## Lesson (1) Factorizing quadratic Trinomial

Factorize each of the following by taking out the highest common factor:

1 3 
$$x$$
 + 21  $y$  = 3 (······· + ·······)

2 
$$2 a^3 + 6 a^2 - 4 a = \cdots (\cdots + \cdots - \cdots)$$

4 
$$(x-5)$$
  $x^2 + (x-5)$   $y^2 = \cdots$ 

**5** 
$$a(a-b)-b(b-a) = \cdots$$

Factorize each of the following:

$$1 x^2 + 5 x + 6$$

$$2 x^2 - 5 x + 6$$

$$3x^2 + 5x - 6$$

$$4x^2-5x-6$$

## Complete each of the following:

- Two numbers such that their product = 30 and their sum is 11
- Two numbers such that their product = 12 and their sum is -8
- $x^2 11 x + 18 = (x \dots) (x \dots)$
- $x^2 + 5x + 6 = (\cdots)(x+2)$
- °.  $(x \dots)$  is a factor of the expression  $x^2 x 6$
- If (x + 2y) = 4 and (x y) = 1, then the numerical value of the expression  $x^2 + xy 2y^2$  is .......
- V. If  $k \in \mathbb{Z}$ ,  $x^2 + kx 3$  can be factorized, then  $k = \dots$
- The rectangle whose area is  $(x^2-7x+6)$  square unit and if its length is (x-6) length unit, then its width is ..... length unit.

Homework

- Two numbers such that their product = -18 and their sum is 3
- Yes Two numbers such that their product = -15 and their sum = -14
- 11.  $x^2 + \cdots + 35 = (x + \cdots + 5)$
- If (x-2) is a factor of the expression  $x^2 8x + 12$ , then the other factor is ........

If (a-b) = 1 and (x + y) = -3, then  $a(x + y) - b(x + y) = \cdots$ ۱٣. If (x-4) is a factor of the expression  $x^2-5$  x+4, then the other factor is ........ ١٤.  $x^2 + \cdots + 8 = (x + 2) (\cdots + \cdots)$ 10 Choose the correct answer: If a - b = 3, then  $6a - 6b = \dots$ ١. (a) 2(b)9(c) 18 (d) 3 The expression :  $x^2 - x - a$  can be factorized if  $a = \cdots$ ۲. (a) 3 (b)4(c) 5(d) 6If  $x^2 - 2x - k = (x + 3)(x - 5)$ , then  $k = \dots$ ٣. (a) - 2(b) - 8(d) 2(c) 15 The expression  $\chi^2 - 3 \chi + c$  can be factorized when  $c = \cdots$ ٤. (a) 1 (c)4(d) 6(b) 2If the expression  $\chi^2 + b \chi - 10$  can be factorized, then b may be ...... ٥. (a)3(b) 2 (c) 1 (d) - 1The number which can be added to the expression :  $x^2 - 11 x + 15$  to be factorized ٦. is ...... (b) 2(c)3(a) 1 (d)4Homework If X - y = 3, X - 2y = 5, then  $X^2 - 3Xy + 2y^2 = \cdots$ ٧. (a) 15 (b) 8 (c)2(d) - 2The expression  $x^2 + 7x + a$  can be factorized if  $a = \cdots$ ۸. (a) 8 (c) 18 (d) 49 (b) 10 For the expression  $x^2 - x - k$  can be factorized then  $k \neq \dots$ ٩. (b) 30 (a) 12 (d) 8 (c) 6If the expression  $x^2 + a x + 2$  can be factorized, then a may be ...... ١٠. (a) 1 (b) 2(c)3(d)4

If the expression  $x^2 - c x + 12$  can be factorized, then c may be ...... (a) - 1(d) 1 (b)4(c)7The number which can be added to the expression  $x^2 - 8x + 5$  to be factorized is ...... ١٢. (c)4(d) 5(a) 1 (b) 2Factorize each of the following perfectly:  $x^2 + 8x + 15$ ١.  $x^2 - 17 x + 30$ ۲.  $x^2 - 6x - 16$  $b^2 + 3 bc - 10 c^2$  $x^2 - 7 x y - 18 y^2$  $15 a + a^2 - 34$  $x^2 + 21 - 10 x$ X(X+7)+10 $(x-1)^2-2(x-1)-8$ Homework  $x^2 + 11 x + 10$ ١..  $x^2 - 7x + 12$ ١١.  $x^2 + 5 x - 14$ ١٢.  $x^2 + 4x - 12$  $x^2 - 3x - 10$ 10.  $l^6 - 6 l^3 - 40$  $x^2 - 4x - 3(x - 2)$ ١٦.

## Lesson (2) Factorizing quadratic Trinomial Follow

## Complete each of the following:

5 
$$y^2 + 16 y + 3 = (5 y + \cdots) (y + \cdots)$$

Y. 
$$5 x^2 - 2 x - 7 = (5 x - \dots) (x + \dots)$$

$$^{\text{r}}$$
.  $3 x^2 + 10 x + 8 = (\dots + 4) (x + \dots)$ 

$$6 x^2 - 11 x - 10 = (2 x - \dots + 2)$$

$$3 x^2 + 7 x - 6 = (3 x - \dots ) (\dots + \dots )$$

If 
$$9 x^2 + 39 x + 36 = 3 (3 x + c) (x + 3)$$
, then  $c = \cdots$ 

V. If 
$$(x + 1)$$
 is a factor of the expression  $2x^2 - x - 3$ , then the other factor is ........

^. If 
$$a(x + y) - b(x + y) = 15$$
 and  $a - b = 3$ , then  $x + y = \dots$ 

9. If 
$$a^2 + k + 6 = (a - 3) (a - 2)$$
, then  $k = \dots$ 

### Homework

$$(x + x - 6) = (x + x - 6)$$

11. 
$$2x^2 - \dots - 2y$$

17. 
$$5 x^2 - 3 x y - \dots = (x - y) (\dots + \dots)$$

$$3a^2 - 5a - 2 = (3a + \cdots) (a - \cdots)$$

16. If 
$$x + 3y = 7$$
,  $x - y = 3$ , then  $x^2 + 2xy - 3y^2 = \dots$ 

### Choose the correct answer:

If 
$$x^2 + a x - 13 = (x + 1) (x - 13)$$
, then  $a = \dots$ 
(a) zero (b) 25 (c) - 12 (d) 12

 $x^2 + 7x + c$  can be factorized if  $c = \cdots$ (b) - 12(a) 12 (c) 17 (d)9The number which can be added to the expression:  $2 x^2 + 5 x - 10$  to be ٣. factorized is ..... (a) - 1(c) - 3(b) -2(d) - 4Homework If  $2 x^2 - c x - 3 = (2 x - 1) (x + 3)$ , then  $c = \cdots$ (a) 5(b) - 5(d) - 7(c)7 $6x^2 - 7x - 3 = \cdots$ (a) (3 X - 1) (2 X - 3)(b) (3 X + 1) (2 X - 3)(c) (3 X + 1) (2 X + 3)(d) (3 X - 1) (2 X + 3)The rectangle whose area is  $(2 x^2 - 3 x - 5)$  cm<sup>2</sup> and one of its dimensions is (X + 1) cm., the second dimension is ......... cm. (c) (2 X + 5)(a) (X - 5)(b) (2 X - 5)(d) (2 X - 3)Factorize each of the following perfectly:  $2 x^2 + 3 x + 1$ ١.  $5z^2 - 7z + 2$ ۲.  $3 x^2 - 14 x - 5$ ٣.  $3 x^2 + 10 x + 8$ ٤.  $8z^2 + 2z - 3$ 

•	$21 X^2 y^2 + 6 X^2 y^3 - 15 X^2 y^4$
	Homework
•	$3 a^2 + 7 a + 2$
	$3x^2 - 10x + 7$
٠.	$5x^2 + 4x - 12$
١.	$8x^2 + 14x + 5$
۲.	$6x^2 - 11x + 3$
٣.	$3y^2 + 7y - 6$
	$25 \text{ m} - 10 + 15 \text{ m}^2$
٤.	

## Lesson (3) Factorizing a perfect square trinomial

### $ilde{\ }$ The perfect square trinomial has the following properties : ullet

- The first term is a perfect square and it is always positive.
- 2 The third term is a perfect square and it is positive also.
- 3 The middle term =  $\pm 2\sqrt{1^{\text{st}} \text{term}} \times \sqrt{3^{\text{rd}} \text{term}}$

#### If the trinomial is a perfect square , then:

- The middle term =  $\pm 2 \times \sqrt{\text{the first term}} \times \sqrt{\text{the third term}}$
- The first term =  $\frac{\text{(the middle term)}^2}{4 \times \text{the third term}}$
- 3 The third term =  $\frac{\text{(the middle term)}^2}{4 \times \text{the first term}}$

### If the trinomial is a perfect square, then we can factorize it to be in the form:

 $(\sqrt{\text{The first term}} \pm \sqrt{\text{The third term}})^2$ 

## Complete to get a perfect square:

- $4x^2 1$
- $9 a^2 \cdots + 36$
- $\frac{1}{25} x^2 \cdots + \frac{1}{4} y^2$
- $18 \text{ } y^2 + 81$
- The value of m which makes the expression :  $4 x^2 + 12 x + m$ , a perfect square is ......

#### Homework

- 7.  $4 a^2 4 a^2 + 36 b^2$
- $z^4 1 + 49 \ell^2$
- $^{\wedge}$  25 m<sup>2</sup> + 20 mn + .....

### Choose the correct answer:

If  $x^2 + k x + 16$  is a perfect square, then  $k = \cdots$ 

(a)4

 $(b) \pm 4$ 

 $(c) \pm 8$ 

(d) 1

If  $x^2 - 2xy + y^2 = 25$ , then  $x - y = \dots$ 

(a) 25

(b) - 5

(c)5

 $(d) \pm 5$ 

 $5 x^2 - 8 x y - 4 y^2 = \cdots$ 

 $^{\text{T}}$ . (a) (5 X + 2 y) (X - 2 y)

(b) (5 X - 2 y) (X + 2 y)

(c) (5 X - 4 y) (X + y)

(d) (X - 4y) (5X + y)

If  $a^2 + b^2 = 11$ , ab = 5, then  $a - b = \dots$ 

(a) 6

 $(b) \pm 1$ 

(c) 1

(d) - 1

The value of c which makes the expression  $c x^2 + 10 x + 1$  a perfect square is .......

(a) 25

٥.

(b) 10

(c)9

(d) 5

If X = 6, y = 4, then  $X^2 - 2Xy + y^2 = \dots$ 

٦. (a) 2

(b) 4

(c) 10

(d) 100

Homework

The expression: a  $x^2 - 40 x + 25$  is a perfect square when a = ......

(a) 2

(b) 4

(c)9

(d) 16

If  $x^2 + kx + 25$  is a perfect square, then  $k = \dots$ 

<sup>^</sup>. (a) 5

٩.

١.,

(b) 10

 $(c) \pm 10$ 

 $(d) \pm 5$ 

If the expression  $x^2 + a x + 16$  is a perfect square, then  $a = \dots$ 

(a) zero

(b)  $\pm 16$ 

 $(c) \pm 4$ 

 $(d) \pm 8$ 

If the expression  $x^2 + 14x + b$  is a perfect square, then  $b = \dots$ 

(a) 2

(b) 7

(c) 14

(d) 49

The value of k which makes the expression  $16 x^2 - 24 x + k$  a perfect square is .......

11. (a) 6

(b) 9

(c) 12

(d) 24

The expression a  $x^2 - 40 x + 25$  is a perfect square when  $a = \dots$ (a) 2 (d) 16 If the expression  $c + 3 x + \frac{1}{4}$  is a perfect square, then  $c = \dots$ ۱۳. (b)  $\frac{9}{4} X^2$ (a) 9 (c)  $9 X^2$ (d)  $4 X^2$ Factorize each of the following perfectly:  $m^2 - 2 m + 1$  $x^2 + 2 x y + y^2$ ۲.  $9a^2 + 6ab + b^2$ ٣.  $36 - 60 \text{ k} + 25 \text{ k}^2$ ٤.  $6a^4 - 12a^2b^2 + 6b^4$  $24 x + 24 x^2 + 6 x^3$  $4 b^2 c + bc^2 + 4 b^3$ ٧.  $(c-d) + 2 X (c-d) + X^{2} (c-d)$ ۸.  $\frac{1}{16}a^2 + \frac{1}{10}a + \frac{1}{25}$  $9 a^2 + 5 b (5 b - 6 a)$  $4 x^2 - 7 y (4 x - 7 y)$ 

### Homework

17. 
$$9x^2 + 12x + 4$$

$$\frac{1}{25}b^2 - 10b + 1$$

15. 
$$4 x^2 - 4 x y + y^2$$

$$18 y^2 - 12 y + 2$$

$$\frac{1}{20}$$
 a y<sup>2</sup> - 60 a y + 45 a

$$3z + 42z^4 + 147z^7$$

14. 
$$0.01 \times^2 - 0.2 \times + 1$$

## Use the factorization to find the value of each:

$$(87)^2 + 2 \times 13 \times 87 + (13)^2$$

$$(997)^2 + 6 \times 997 + 9$$

### Homework

$$(7.3)^2 + 2 \times 7.3 \times 2.7 + (2.7)^2$$

$$(20.7)^2 - 1.4 \times 20.7 + (0.7)^2$$



## Lesson (4) Factorizing the difference of two squares

The difference of two squares of two quantities

= (the sum of the two quantities) × (the difference of the two quantities)

$$a^2 - b^2 = (a + b) (a - b)$$

## Complete each of the following:

	1 0 0
١.	$(2 X + \cdots - 3 y) = 4 X^2 - \cdots$
۲.	$(\cdots + 3 \text{ m}) (\cdots - 3 \text{ m}) = 25 \times 2 - \cdots$
٣.	If $a - b = 2$ , $a + b = 3$ , then $a^2 - b^2 = \dots$
٤.	If $x^2 - y^2 = 20$ , $x + y = 10$ , then $x - y = \dots$
٥.	If $x^2 - y^2 = x + y$ , then $x - y = \dots$
٦.	If $a + b = 7 (a - b) = 14$ , then $a^2 - b^2 = \cdots$
٧.	If $(39)^2 - 1 = 40  x$ , then $x = \dots$
۸.	$\frac{1}{2} x^2 - 2 = \frac{1}{2} (\cdots ) (\cdots )$
	Homework
٩.	$\cdots - 64 \chi^2 = (4 - \cdots) (4 + \cdots)$
١٠.	If $a^2 - b^2 = 45$ , $a - b = 5$ , then $\sqrt{a + b} = \dots$
١١.	If 2 (a - b) (a + b) = 18, then $a^2 - b^2 = \cdots$
۱۲.	If $x + y = 5$ , $x - y = 1$ , then $x^2 - y^2 = \dots$
۱۳.	If $x + y = 3 (x - y) = 12$ , then $x^2 - y^2 = \dots$
١٤.	$(75)^2 - (25)^2 = 100 \times \dots$
10.	$3 x^2 - 5 x - 2 = (3 x + \dots - 2)$

### Choose the correct answer:

If  $X^2 - a = (X - 3)(X + 3)$ , then  $a = \dots$ 

(a) 3

١.

٣.

**(b)** -3

(c)9

(d) - 9

If  $x^2 + \ell - 4 = (x - 2)(x + 2)$ , then  $\ell = \dots$ 

- (a) zero
- (b) 2

(c)4

(d) 8

If a - b = 7, a + b = 5, then  $2a^2 - 2b^2 = \dots$ 

- (a) 2
- (b) 12

(c) 35

(d) 70

If  $x^2 - y^2 = 16$ , y - x = 2, then  $x + y = \dots$ 

- (a) 4
- (b) 8

(c) - 8

(d) 2

If  $(25)^2 - (15)^2 = 10 \ x$ , then  $x = \dots$ 

- (a) 40
- (b) 30

(c) 20

(d) 10

 $(X - y) (X + y) (X^4 - 2 X^2 y^2 + y^4) = \cdots$ 

7. (a)  $x^6 - y^6$ 

(b)  $(X - y)^3 (X + y)^3$ 

(c)  $(X^3 - y^3)(X^3 + y^3)$ 

(d)  $(X^2 + y^2) (X^2 - y^2)$ 

If a + b = 8, b - a = -5, then  $a^2 - b^2 = \dots$ 

(a) - 40

(b) 40

(c) 13

(d) - 13

Homework

If x + 2y = 3,  $x^2 - 4y^2 = 21$ , then  $x - 2y = \dots$ 

- (a) 14
- (b) 9

(c) 7

(d) 6

If  $x^2 - y^2 = 24$ , x + y = 8, then  $3x - 3y = \dots$ 

(a)  $\frac{1}{3}$ 

٩.

(b) 3

(c) 9

(d) 16

If a + b = 5, a - b = 4, then  $b^2 - a^2 = \dots$ 

- (a) 20
- (b) 1

(c) 9

(d) 20

If  $x^2 - y^2 = 16$ , x + y = 8, then  $x - y = \cdots$ 

'· (a)

(b) 1

(c) 128

(d) 64

If the expression:  $x^2 + 7x + k$  can be factorized, then  $k = \dots$ ١٢. (a) 16 (b) - 12(d)6(c) 30 The expression:  $4 x^2 + k + 25 y^2$  is a perfect square when  $k = \dots$ ۱٣. (a) 20 (c)  $20 \times y$  $(d) \pm 20 X y$ (b)  $10 \times y$  $x^2 - \dots = (x-7)(x+7)$ 1 ٤ (a)7(d) - 7(c) - 49(b) 49 If  $x^2 + 2xy + y^2 = 9$ , then  $x + y = \dots$ (a) 9 $(c) \pm 3$  $(d) \pm 9$ (b) 3Factorize each of the following perfectly:  $x^2 - 4$ ١.  $225 X^2 - y^2$ ۲.  $625 a^2 - 81 b^2$  $9 - y^2$ ٤.  $a^2 - b^2 c^4$  $\frac{1}{9} y^2 - 2\frac{1}{4}$ ٦.  $0.04 \, x^2 - 0.25 \, y^2$ ٧.  $x^4 - 16 y^4$  $8 x^2 - 50$  $27 x^3 - 48 x y^6$  $\frac{1}{2} x^2 - \frac{1}{18} y^2$ ١١. 17.  $3 x^2 - \frac{3}{16}$ 

Homework

1°. 
$$a^2-25$$

17. 
$$-9x^2+25$$

$$10^{1} \times 10^{1} \times 1$$

14. 
$$\frac{a^2}{25} - \frac{4b^2}{49}$$

$$19. \quad \mathbf{X}^4 - 1$$

$$\forall \cdot . \quad \left| \frac{1}{3} x^2 - 3 \right|$$

## Use the factorization to find the value of each:

 $(77)^2 - (23)^2$ 

 $(75)^2 - (25)^2$ 

 $r_{.}$   $(95)^2 - 25$ 

### Homework

 $\epsilon$ .  $(78)^2 - (77)^2$ 

 $\circ$ .  $(999)^2 - 1$ 

#### L6220U (A)

## Factorizing the sum and the difference of two cubes

The sum of two cubes of two quantities =

(the first + the second) (the square of the first - the first  $\times$  the second + the square of the second)

**i.e.** 
$$a^3 \oplus b^3 = (a + b) (a^2 \ominus ab + b^2)$$

The difference between two cubes of two quantities =

(the first – the second) (the square of the first + the first  $\times$  the second + the square of the second)

**i.e.** 
$$a^3 \ominus b^3 = (a - b) (a^2 \oplus ab + b^2)$$

### Complete each of the following:

$$x^3 - 1 = (x - 1)$$
 (.....)

$$\chi^{12} + y^{15} = (\dots + \dots) (\dots + \dots)$$

- Y. If x-3 is a factor of the expression  $x^3-27$ , then the second factor is ........
- i. If x + y = 2,  $x^2 xy + y^2 = 8$ , then  $x^3 + y^3 = \dots$ 
  - If  $(a + b)^2 = 16$ ,  $a^2 + b^2 = 8$ , then 2 a b = .......
- ٥.

### Homework

$$8 a^3 + 125 = (\cdots + \cdots) (4 a^2 - 10 a + \cdots)$$

$$\vee$$
.  $8 a^3 - \dots = (\dots - \dots + 9)$ 

 $\wedge$ . If 4  $a^2 - 2a + 1$  is a factor of the expression 8  $a^3 + 1$ , then the other factor is .........

9. 
$$2 x^2 - 7 x - 15 = (2 x + 3) ( \dots )$$

If 
$$k x^2 + 4 x + 1$$
 is a perfect square, then  $k = \dots$ 

11. If 
$$(x+2)$$
 is a factor of the expression :  $x^2-x-6$ , then the other factor is ........

If 
$$x^2 + a x + 5$$
 can be factorized, then  $a = \cdots$ 

### Choose the correct answer:

If 
$$x^3 - y^3 = 14$$
,  $x^2 + xy + y^2 = 7$ , then  $x - y = \dots$ 

- (a) 2
- (b) 7

- (c) 14
- (d) 2

If 
$$y^3 - a = (y - 2) (y^2 + 2 y + 4)$$
, then  $a = \dots$ 

- (a) 2
- (b) 4

(d) - 8

If 
$$x^3 + 27 = (x + 3)(x^2 + k + 9)$$
, then "k" equals ......

- (a) 6 X
- (b) -3 X
- (c) 3 X
- (d) 6 X

$$(x-y)(x+y)(x^4+x^2y^2+y^4)=\cdots$$

- ٤. (a)  $\chi^3 - y^3$  (b)  $\chi^3 + y^3$  (c)  $\chi^6 - y^6$
- (d)  $x^6 + y^6$

If 
$$a^2 + b^2 = 11$$
,  $ab = 5$ , then  $a - b = \cdots$ 

(a) 6

٦.

 $(b) \pm 1$ 

(c) 1

(d) - 1

If 
$$a - b = 5$$
, then  $a^2 - 2ab + b^2 = \cdots$ 

(a) 25

(b) 20

(c) 15

(d) 10

### Homework

If 
$$x + y = 3$$
,  $x^2 - xy + y^2 = 5$ , then  $x^3 + y^3 = \dots$ 

- (a) 15
- (b) 25

(d)7

If 
$$x^3 + y^3 = 28$$
,  $x + y = 2$ , then  $x^2 - xy + y^2 = \dots$ 

- (a) 28
- (b) 14

(c) 2

(d) 7

If 
$$x^3 - 8 = (x + a)(x^2 + 2x + 4)$$
, then  $a = \dots$ 

- (a) 4
- (b) -4
- (c) 2

(d) - 2

$$x^3 - k^3 = (x - k)(x^2 + 4x + k^2)$$
, then  $k = \dots$ 

١٠. (a) 2

(b) 4

- (c) 16
- (d) 64

$$x^3 + 8 = (x + 2) (\cdots )$$

- (a) x-2 (b)  $x^2+2x+4$  (c)  $x^2-4x+4$  (d)  $x^2-2x+4$

If 
$$x^3 + 8 = (x + 2)(x^2 + a + 4)$$
, then  $a = \dots$ 

(a) X

(b) - X

- (c)-4X
- (d)-2X

If 
$$x^2 + e - 16 = (x + 4)(x - 4)$$
, then  $e = \cdots$ 

(b) zero

- $(c) 8 \chi$
- (d) 4 X

	$(x^3 + 64) \div (x + 4)$	=				
١٤.	$(a) x^2 + 16$ (b) x			$x^2 - 4x + 16$		
	(c) $x^2 + 4x + 16$ (d) $x^2 - 4x - 16$					
10.	$3 x^2 y + 6 x y = \cdots$	$3 x^2 y + 6 x y = \cdots (x+2)$				
	(a) 3 X	(b) 3 X y <sup>2</sup>	(c) $X^2$ y	(d) 3 X y		
١٦.	$(64)^2 - (36)^2 = \cdots$ (a) 100					
' '.	(a) 100	(b) 28	(c) 2800	(d) 280		
F	actorize each	h of the foll	lowing perf	ectly:		
١.	$8 x^3 - 125$					
۲.	$m^3 + 64 n^3$					
٣.	$\frac{1}{8} a^3 - 8 b^3$					
١.						
٤.	$0.027 \text{ m}^3 - \text{n}^3$					
٥.	$8 x^3 - 343 y^6$					
٦.	$16 a^3 b + 686 b^4$					
`.						
	$x^6 - 7x^3 - 8$					
٧.						
	If $X^2 - y^2 = 20$ , $X - y = 2$ , $X^2 - Xy + y^2 = 28$					
۸.	Find the value of $x^3 + y^3$					
	Factorize the following expression perfectly: $(x^3 - 9)(x^3 + 9) + 17$					
٩.						

Homework

١٠.	$\chi^3 + 8$
١١.	$x^3-1$
١٢.	$512  \chi^3 - y^3$
۱۳.	$\ell^3 \text{ m} - 27 \text{ m}^4$
	$m^6 + 7m^3 - 8$
١٤.	



## Lesson (0) Factorizing by grouping

## Factorize each of the following perfectly:

	a X + b X + a y + by
١.	
	am - an + m - n
۲.	
-	2 2
	$a^2 + 2ab + b^2 - c^2$
٣.	
	Homework
	xy + 5y + 7x + 35
٤.	
	5 l - 10 m - a l + 2 am
٥.	

	$9 x^2 - 4 a^2 + y^2 + 6 x y$			
٦.				
	$ab X^2 + b X - a X - 1$			
٧.				
	$25x^2 - 30x + 9 - 16y^2$			
۸.				
	$x^2 - 9a^2 + y^2 + 2xy$			
٩.				

## Lesson (/) Factorizing by completing the square

### - The method of factorization by completing the square : •

- We add to the given expression twice the product of the two square roots of the two perfect square terms and subtract it again not to change the main expression.
- 2 Using the commutative and associative properties, we rewrite the expression after ordering its terms to get the form:

a perfect square trinomial - a perfect square monomial

- 3 We factorize the resultant expression as a difference between two squares.
- 4 If it is possible, we should factorize the resultant expressions (resultant factors) in order that the factorization is perfect.

## Factorize each of the following perfectly:

١.	$x^4 + 4y^4$
۲.	$a^4 + 2500 b^4$
	$8 x^4 y^2 + 162 z^4 y^2$
٣.	

	$x^4 + 9x^2 + 81$			
٤.				
	$m^4 - 11 m^2 n^2 + n^4$			
٥.				
	$4 x^4 + 25 y^4 - 29 x^2 y^2$			
٦.				
	Homework			
	$81 \ x^4 + 4 \ z^4$			
٧.				
	$4 x^4 + 625 z^4$			
۸.				

	$9 x^4 - 25 x^2 + 16$
٩.	
	$x^4 + x^2 y^2 + 25 y^4$
١٠.	
	$16 x^4 - 28 x^2 y^2 + 9 y^4$
١١.	

### Lesson (o)

## Solving quadratic equations in one variable algebraically

## Complete each of the following:

If -5 is a root of the equation :  $x^2 + 2x - 15 = 0$ ١.

, then the other root is ......

If x = 2 is a root of the equation :  $x^2 - 6x + k = 0$ , then  $k = \dots$ ۲.

and the other root is ......

If one of the roots of the equation :  $2 x^2 + 8 x = 0$ ٣.

is a root of the equation :  $x^2 + 5x + a = 0$ , then  $a = \dots$  or .......

### Homework

If the number 9 is a solution of the equation :  $x^2 + k = 0$ , then  $k = \dots$ ٤.

The solution set of the equation :  $x^2 + 25 = 0$  in  $\mathbb{R}$  is ...... ٥.

The solution set of the equation  $x^2 = 4 x$  in  $\mathbb{R}$  is ...... ٦.

### Choose the correct answer:

The S.S. of the equation: 3(x-2)(x+5) = 0 in  $\mathbb{R}$  is .....

(a)  $\{0,2,-5\}$  (b)  $\{3,2,-5\}$  (c)  $\{2,-5\}$  (d)  $\{-2,5\}$ 

The S.S. of the equation :  $\chi^2 - 4 = 0$  in  $\mathbb{R}$  is ......

۲. (a)  $\{4\}$ 

١.

٣.

(b)  $\{4,-4\}$  (c)  $\{2\}$  (d)  $\{2,-2\}$ 

The S.S. of the equation :  $x^2 + 25 = 0$  in  $\mathbb{R}$  is .......

(a)  $\{5\}$ 

(b)  $\{5, -5\}$  (c)  $\{-5\}$ 

(d) Ø

The equation whose roots are 3 and 5 is ......

(a)  $5 X^2 + 8 X + 3 = 0$ ٤.

(b)  $2 x^2 + 8 x - 15 = 0$ 

(c)  $\chi^2 - 8 \chi + 15 = 0$ 

(d)  $3 x^2 + 8 x + 5 = 0$ 

The S.S. of the equation : X(X-3) = 5 X in  $\mathbb{R}$  is .....

(a)  $\{3\}$ 

(b)  $\{0,3,5\}$  (c)  $\{3,5\}$  (d)  $\{0,8\}$ 

٦.	The S.S. of the equation : $\frac{4}{x} = \frac{x}{9}$ in $\mathbb{R}$ is				
	(a) {4,9}	(b) $\{6, -6\}$	(c) {6}	(d) $\{36\}$	
٧.	If the number 4 is a solution of the equation : $x^2 + x - 20 = 0$ , then the other solution is				
	(a) 20	(b) 5	(c) – 5	(d) - 4	
		Homewor	·k		
	The S.S. of the equ	uation: $(x-4)^2 = 0$ in $\mathbb{R}$	is		
١.	(a) {4}	(b) $\{0,4\}$	(c) $\{0, -4\}$	$(d) \left\{-4\right\}$	
1	The solution set of	f the equation : $\mathcal{X}(X-3)$	) = 0 in R is		
١.	(a) {3}	(b) {0,3}	(c) $\{0, -3\}$	(d) {0}	
١٠.	$\int 1 \cdot 1 $	= (3 X - 2) (X + 3), then	c =		
••	(a) 7	(b) 12	(c) 13	(d) 5	
١.	The expression: 2	$x^2 + 6x + a$ is a perfect	square when a = ·····		
٠.	(a) 6	(b) 16	(c) 1	(d) 9	
۲.	$x^3 + y^3 = (\cdots ) (x^2 - xy + y^2)$ (a) $x^2 + y^2$ (b) $x^2 - y^2$ (c) $x + y$ (d) $x - y$				
٠.	(a) $x^2 + y^2$	(b) $\chi^2 - y^2$	(c) X + y	(d) X – y	
ų.	One of the factors	of the expression : $\chi^2$ –	3 X – 18 is ·······		
۱۳.	(a) $X-3$	(b) $X - 6$	(c) $X - 9$	(d) $X - 18$	
F	ind in R the so	lution set of each	of the following	ng equations:	
	$x^2 - 7x - 30 = 0$				
١.					
	$2x^2 + 7x = 0$				
•					
٣.	$(x+2)^2 = 25$				

	(x-3)(x+5) = 20
٤.	
	$\chi - \frac{2}{\chi} = \frac{7}{2}$
٥.	
	X(X-1)=6
٦.	
	$3 x^3 = 12 x$
٧.	
	$x^3 - 4x = 0$
۸.	
	$x^4 - 13 x^2 + 36 = 0$
٩.	
٩.	

١١.	If: $x + \frac{1}{x} = 2$ , then <b>find</b> : $x^2 + \frac{1}{x^2}$
	I tomovioule
	Homework
	$\chi^2 - 5 \chi - 6 = 0$
١٢.	
١١.	
	$x^2-6x=-9$
۱۳.	
	3 _
	$x - \frac{3}{x} = 2$
١٤.	
	$x^2 - 5 x = 0$
١٥.	
	$4 x^2 = 25$
	4 A = 23
١٦.	

#### Lesson (y)

### Applications on solving quadratic equations in one variable algebraically

## Complete each of the following:

١.	Twice the square of the number $X$ is
----	---------------------------------------

If the age of Bassim now is X years, then his age 3 years ago was ...... years. ۲.

Choose the correct answer: If the age of Ayman 5 years ago was X years, then the square of his age now = ....... ١. (d)  $(x-5)^2$ (b)  $x^2 + 25$ (c)  $(x+5)^2$ (a)  $X^2 + 5$ If the age of Bassim now is X years, then his age 3 years ago was ...... years. ۲. (d)  $X^3$ (a) 3 X(b) X + 3(c) X - 3If the age of Amgad now is X years, then his age after 7 years will be ....... years. ٣. (d)  $\chi^7$ (c) X + 7(a) 7 X(b) X - 7If the age of Ayman 5 years ago was x years, then his age now is ...... years. ٤. (d)  $\frac{x}{5}$ (a) X-5(b) X + 5(c) 5 XIf the age of Sally 2 years ago was X years, then her age after 3 years from now will be ..... years. ٥. (d) 6X(a) X + 2(b) X + 3(c) X + 5If the age of Magdy now is X years, then the square of his age after 2 years is ........ ٦. (a)  $X^2 + 2$ (b)  $X^2 + 4$ (c)  $(x-2)^2$ (d)  $(x + 2)^2$ If the age of Samy now is X years, then twice his age 5 years ago is ......... years. ٧. (a) X - 5(c) X - 10(b) 2 X - 5(d) 2 X - 10Three times the square of the number X is ....... ۸. (d)  $\frac{\chi^2}{2}$ (b)  $x^2 + 3$ (c)  $3 X^2$ (a)  $(3 X)^2$ 

# Essay problems: A positive integer whose square is more than five times the number by 36 Find the number. An integer , if we add twice its square to the number 7 the result will be 135 Find the number. ۲. Find the real number whose double exceeds its multiplicative inverse by one. ٣. Find two real numbers whose product is 45 and one of them is 4 more than the other. ٤.

	The sum of the squares of two successive odd numbers is 130			
	Find the two numbers.			
٥.				
	The sum of three successive integers is equal to the square of their middle integer.			
	Find these numbers.			
٦.				
	Hatem is 4 years older than Hanan now, and the sum of squares of their ages now is 26			
	Find their ages now.			
٧.				
	A right-angled triangle, the lengths of the two sides of the right angle are $4 \times cm$ .			
	and $X + 1$ cm. If the area of the triangle = 84 cm <sup>2</sup> , calculate the length of its			
	hypotenuse.			
۸.				

## Homework

Find the rational number whose four times its square equals 81  What is the real number if it is added to its square, the result will be 12?  Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm?	٩.	What is the real number which exceeds its multiplicative inverse by $\frac{5}{6}$ ?			
Find the rational number whose four times its square equals 81  What is the real number if it is added to its square, the result will be 12?  Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm.					
What is the real number if it is added to its square, the result will be 12?  Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm?					
What is the real number if it is added to its square, the result will be 12?  Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm?					
What is the real number if it is added to its square, the result will be 12?  Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm.	١٠.	Find the rational number whose four times its square equals 81			
What is the real number if it is added to its square, the result will be 12?  Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm.					
Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm <sup>2</sup>					
Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm <sup>2</sup>					
Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm <sup>2</sup>	١١.	What is the real number if it is added to its square, the result will be 12?			
Find the dimensions of a rectangle whose length is 4 cm. more than its width and whose area is 21 cm <sup>2</sup>					
whose area is 21 cm <sup>2</sup>					
whose area is 21 cm <sup>2</sup>					
	۱۲.				

## Lesson (1U) Integer powers in R

### → Non-negative integer powers in R •

If  $a \in \mathbb{R}$ ,  $n \in \mathbb{Z}^+$ , then  $a^n = a \times a \times a \times a \times a \times a \times \cdots \times a$  where a is repeated as a factor n times. The symbol  $(a^n)$  is read as: a to the power n or the  $n^{th}$  power of the number a or the base a

### 🗕 Negative integer powers in 🏿 🕳

If a is a real number,  $a \neq 0$  and n is a positive integer, then:

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad a^n = \frac{1}{a^{-n}}$$

If  $a \in \mathbb{R}^*$  (The set of non-zero real numbers), then:  $a^0 = 1$ 

$$(-a)^n = a^n$$
 if n is an even number

$$(-a)^n = -a^n$$
 if n is an odd number

#### Remarks

For every  $a \in \mathbb{R}^+$ ,  $n \in \mathbb{Z}^+$ , then  $a^n \times a^{-n} = a^n \times \frac{1}{a^n} = 1$  (the multiplicative neutral)

i.e. an and an are the multiplicative inverse of each other.

2 For every  $a \in \mathbb{R}^*$ ,  $b \in \mathbb{R}^*$  and  $n \in \mathbb{Z}^+$ , then  $\left[\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$ For example:  $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$ 

## Complete each of the following:

Y. If 
$$(x-5)^{zero} = 1$$
, then :  $x \in \dots$ 

$$\Upsilon$$
. If  $a = 7^{x}$  and  $b = 7^{-x}$ , then :  $a \times b = \dots$ 

4. If 
$$x = (\sqrt{2} + 3)^5$$
 and  $y = (\sqrt{2} + 3)^{-5}$ , then:  $x = x = 3$ 

$$\circ. \qquad \left(\frac{5}{6}\right)^{-4} = \left(-\frac{\dots}{\dots}\right)^2$$

7. 
$$\left| \text{ If } \left( \frac{1}{2} \right)^{x} = 5 \right|$$
, then :  $(8)^{-x} = \dots$ 

V. If 
$$2^x = 7$$
,  $2^y = 5$ , then :  $2^{x+y} = \dots$ 

$$^{\wedge}$$
. If  $5^{x} = 3$ ,  $5^{-y} = 7$ , then:  $5^{x+y} = \dots$ 

## Choose the correct answer:

$$\begin{array}{c} 5^2 + 5^2 = \cdots \\ \text{(a) } 10^2 \end{array}$$

(b) 
$$10^4$$

(c) 
$$5^4$$

$$3^5 \times 2^5 = \dots$$

(a) 
$$5^{10}$$

(b) 
$$6^{10}$$

(c) 
$$6^5$$

(d) 
$$6^{25}$$

$$(5 \text{ a})^{\text{zero}} = \cdots , \text{ a} \neq 0$$

3 
$$\chi^{\text{zero}} = \dots, \chi \neq 0$$

(d) 
$$3 X$$

$$3^{(2^3)} = \dots$$

(a) 
$$3^6$$

(b) 
$$3^5$$

(c) 
$$3^8$$

(d) 
$$3^{32}$$

$$\square 4^3 + 4^3 + 4^3 + 4^3 = \dots$$

(a) 
$$4^3$$

(b) 
$$4^4$$

(c) 
$$4^{12}$$

(d) 
$$4^{81}$$

#### The quarter of the number $4^{20} = \cdots$ ٧.

(a) 
$$1^{20}$$

(b) 
$$4^{19}$$

(c) 
$$4^{16}$$

(d) 
$$4^5$$

#### 4 times the number $2^8 = \cdots$ ٨.

(a) 
$$2^{32}$$

(b) 
$$8^8$$

(c) 
$$2^{10}$$

(d) 
$$4^8$$

$$\begin{cases} (\sqrt{3})^6 \times 3^4 = \dots \\ (a)(\sqrt{3})^{24} \end{cases}$$
 (b)  $3^{10}$ 

(a) 
$$\left(\sqrt{3}\right)^{24}$$

(b) 
$$3^{10}$$

(c) 
$$3^7$$

(d) 
$$\left(\sqrt{3}\right)^{10}$$

The value of: 
$$2^{20} + 2^{21} = \dots$$

(a) 
$$2 \times 2^{40}$$

(b) 
$$2 \times 2^{41}$$

(c) 
$$3 \times 2^{20}$$

(d) 
$$3 \times 2^{21}$$

١١.	$\square$ What of the following is closest to $11^2 + 9^2$ ?			
		(b) 211 + 29		(d) 120 + 80
١٢.	$\square$ If $5^{x} = 4$ , then:	5 <sup>X-1</sup> = ·······		-
	(a) 1.25	(b) 0.8	(c) 0.125	(d) 0.08
	$\square 0.002 \times 0.05 = \cdots$			
۱۳.	(a) $10^{-5}$	(b) 10 <sup>-4</sup>	(c) 10 <sup>4</sup>	(d) 10 <sup>5</sup>
١٤.	$\begin{array}{c} \boldsymbol{\mathcal{X}}^{m-1} \times \dots = 1 \\ \text{(a) } \boldsymbol{\mathcal{X}}^{m-1} \end{array}$	$, \chi \neq 0$		
			(c) $X^{m+1}$	(d) $X^{-m+1}$
10	$5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2$ (a) $5^3$	$2 \times 2 \times 2 = 4 \times \dots$		
, • •	(a) $5^3$	(b) $2^3$	(c) $10^3$	(d) $5^3 + 2^3$
	1	Homewoo	rk	
17	$(5^2)^3 = \cdots$ (a) $5^6$			
	(a) $5^6$	(b) 5 <sup>5</sup>	(c) $5^{32}$	(d) 5
•	$2^5 + 2^5 + 2^5 + 2^5 = 0$ (a) $2^4$			
1 4.	(a) 2 <sup>4</sup>	(b) 2 <sup>6</sup>	(c) $2^7$	(d) 2 <sup>20</sup>
۱۸.	☐ Sixth the number	$r 2^{12} \times 3^{12}$ is		
	(a) $6^2$	(b) 6 <sup>4</sup>	(c) 6 <sup>11</sup>	(d) $6^{23}$
۱٩.	Fifth the number $(\sqrt[3]{5})^6$ is			
			(c) 5 <sup>6</sup>	(d) 5 <sup>12</sup>
L.	$\square$ The value of: $2^5$ (a) $2^6$	$+(\sqrt{2})^{10} = \dots$		
۲۰.	(a) $2^6$	(b) $2^{10}$	(c) $\left(\sqrt{2}\right)^{15}$	(d) $\left(\sqrt{2}\right)^{20}$
	If $6^{x} = 11$ , then $6^{x}$	+ <sup>1</sup> =		
۲١.	(a) 12	(b) 22	(c) 66	(d) 72
	If $x = \frac{\sqrt{9}}{\sqrt{3}}$ , then  (a) $\frac{\sqrt{3}}{3}$	<b>~</b> -1_	` '	` '
۲۲.	$\sqrt{3}$ , then			
	(a) $\frac{\sqrt{3}}{3}$	$(b) \frac{\sqrt{3}}{\sqrt{2}}$	$(c)\sqrt{3}$	(d) 2
		Ψ2		

 $\square \left(\sqrt{3} + \sqrt{2}\right)^9 \left(\sqrt{3} - \sqrt{2}\right)^9 = \dots$ 

(a) 1

 $(c)\sqrt{6}$ 

(d) 5

The numerical value of the expression:  $\frac{2^{2n+1} \times 5^{2n+1}}{10^{2n}}$  is ...... 7 £

(a)  $\frac{1}{10}$ 

(b)7

(c) 10

(d) 100

 $2^{2011} = 2^{2010} + \dots$ 

(a) 2

(b) 2010

(c)  $2^{2010}$ 

(d)  $2^{2011}$ 

Find the value of each of the following in the simplest form:

 $3^{-2}$ 

 $\left(\sqrt{5}\right)^4$ 

 $\left(-\sqrt{3}\right)^{-2}$ ٣.

 $(0.01)^{-2}$ 

 $(x^2)^{-3} \times (x^{-3})^{-2}$ 

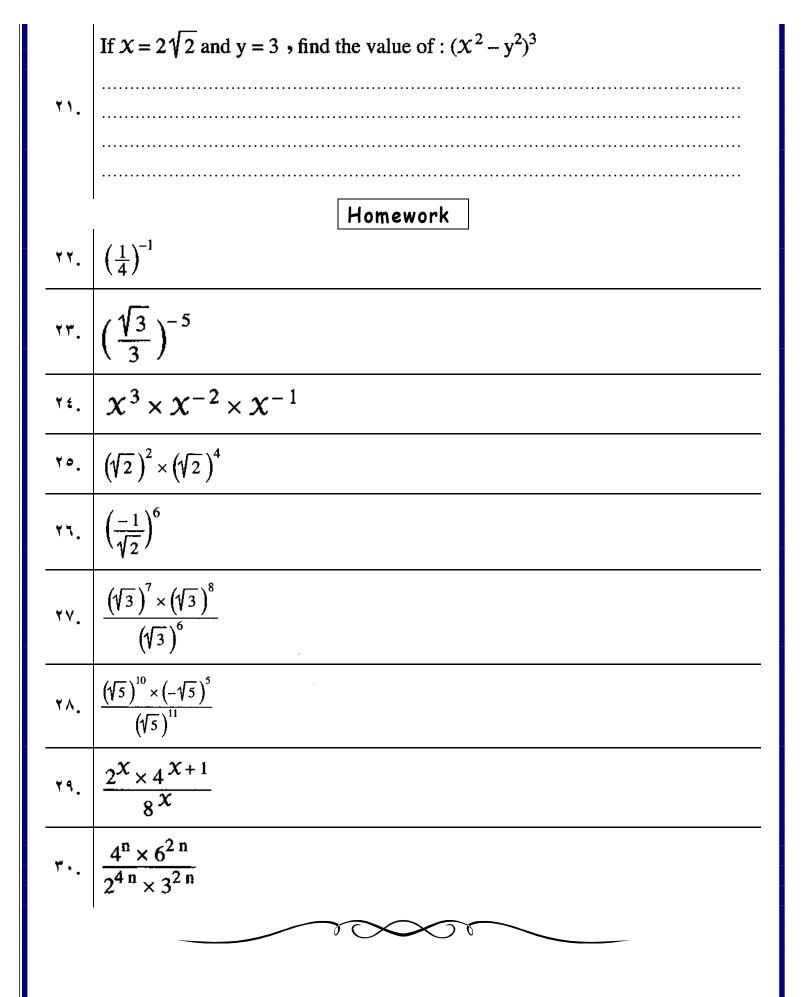
 $\frac{\left(\chi^2\right)^{-3}\times\left(\chi^{-1}\right)^2}{\chi^{-3}\times\chi^{-4}}$ 

 $\left| \left( -\sqrt{5} \right)^9 \div \left( -\sqrt{5} \right)^5 \right.$ 

 $\left(\left(\sqrt{2}\right)^3 \times \left(-\sqrt{2}\right)^2\right)^2$ 

9.  $\left(\sqrt{3}\right)^{-4} \times \left(-\sqrt{2}\right)^{4}$ 1.  $\left(\left(-5\right)^{3}\right)^{2} \times \left(-\sqrt{5}\right)^{-4}$ 

١١.	$\frac{\left(\sqrt{7}\right)^{-4}\times\left(\sqrt{7}\right)^{-3}}{\left(\sqrt{7}\right)^{-9}}$
١٢.	$\frac{\left(\sqrt{3}\right)^5 \times \left(\sqrt{3}\right)^4}{\left(\sqrt{3}\right)^3 \times 27}$
۱۳.	$\frac{(10)^2 \times (10)^{-7}}{(0.1)^2 \times 0.001}$
١٤.	$\left(\frac{3\sqrt{2}}{2\sqrt{3}}\right)^4$
١٥.	$\frac{9^{x}\times3^{x+2}}{(27)^{x}}$
١٦.	$\frac{(36)^{n} \times 5^{2 n}}{(30)^{2 n}}$
١٧.	$\frac{8^{n-1} \times 32^{-n}}{32 \times 4^{-n}}$
١٨.	$\frac{6^{n} \times 4^{n + \frac{1}{2}}}{(24)^{n}}$
۱٩.	If $\frac{8^{x} \times 9^{x}}{18^{x}} = 64$ , find the value of $4^{-x}$
۲٠.	If $a = \sqrt{3}$ and $b = \sqrt{2}$ , find the value of: $a^4 - b^4$ $2 \frac{a^4}{b^4}$



#### Lesson (11) Solving exponential equations in R

If a is a real number, m and n are two integers

and  $a^m = a^n$ , then m = n where :  $a \neq 0$ ,  $a \neq \pm 1$ 

For example:

If  $3^n = 9$ , then:  $3^n = 3^2$ 

, :: the base = the base

 $\therefore$  the power = the power

 $\therefore \mid \mathbf{n} = 2$ 

If a and b are two real numbers, m is an integer and  $a^m = b^m$ , then:

• [a = b] if m is an odd number. For example: If  $n^5 = 3^5$ , then: n = 3

•  $a = \pm b$  if m is an even number. For example: If  $n^2 = 3^2$ , then:  $n = \pm 3$ 

• m = zero if  $a \neq \pm b$ 

**For example:** If  $7^{n-2} = 5^{n-2}$ , then: n-2 = 0 : n = 2

## Complete each of the following:

If  $5^{X(X-1)} = 1$ , then the value of  $X = \dots$ ١.

If  $3^n \times 3^5 = 1$ , then n = ...۲.

٣.

If  $3^{x} + 3^{x} + 3^{x} = 1$ , then x = ......

If  $\{3, a^{X-2}\} = \{1, 3\}$ , then the value of  $X = \dots$ 

If  $(2^x, 125) = (16, y^3)$ , then  $x = \dots$  and  $y = \dots$ 

Homework

If  $2^{y} \times 5^{y} = 100$ , then  $y = \dots$ 

If  $\left(\frac{3}{5}\right)^{\chi-7} = 1$ , then  $\chi = \dots$ 

## Choose the correct answer:

If  $3^{X+1} = 5^{X+1}$ , then  $X = \dots$ 

(a) 4

١.

(b) 3

(c) - 1

(d) 1

If  $3^{2+x} = 5^{x+2}$ , then  $7^{x+2} = \dots$ 

(a) 7

(b) - 7

(c) - 14

(d) 1

If  $\left(\frac{2}{3}\right)^9 = \left(\frac{3}{2}\right)^x$ , then  $x = \dots$ 

(a) - 9

(b) 9

(c) 32

(d) 23

If  $5^{|X-3|} = 25$ , then  $X = \dots$ 

٤.

(a) 5

(b) 2

(c) 1

(d) 5 or 1

 $\square$  If  $2^{x-1} \times 3^{1-x} = \frac{9}{4}$ , then  $x = \dots$ 

(a)-3

(b) - 1

(c) 1

(d) 3

Homework

If  $2^{x} = \frac{1}{8}$ , then  $x^{2} = \dots$ 

(a)  $\frac{1}{4}$ 

(b) 9

(c) - 9

 $(d) - \frac{1}{9}$ 

If  $2^{X-2} = 2^{1-2X}$ , then  $X = \dots$ 

٧.

(a) 2

(b)  $\frac{1}{2}$ 

(c) 1

(d) zero

If  $3^{x} = 9$ , then  $2^{x} - 1 = \dots$ 

٨.

(a) 7

(b) 3

(c) 8

(d) 5

If  $3^{x} = 7$ ,  $7^{y} = 9$ , then  $xy = \dots$ 

٩.

(a) 5

(b) 2

(c) 7

(d) 9

## Essay problems:

Find	the	value	of n	in	each	of	the	followin	ıg	when	n	$\in \mathbb{Z}$	:
------	-----	-------	------	----	------	----	-----	----------	----	------	---	------------------	---

$3^{n-2}=81$

$$\left(\sqrt{3}\right)^{n-1} = 9$$

$$\left(\frac{3}{5}\right)^{n+2} = \frac{125}{27}$$

$$\left(\frac{2}{3}\right)^{n-4} = 2\frac{1}{4}$$

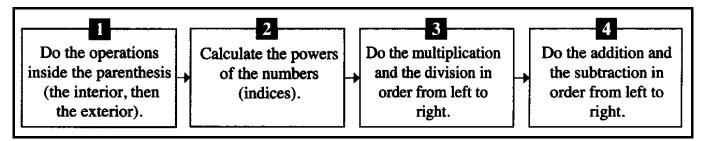
$$\left(\frac{2}{3}\right)^{n+5} = \left(3\frac{3}{8}\right)^{-2}$$

	$2^n \times 9^{n+1}$
	$\frac{2^{n} \times 9^{n+1}}{(18)^{n}} = 3^{n}$
	(10)
_	
٦.	
	(12) <sup>n-1</sup>
	$\frac{(12)^{n-1}}{2^{n-1} \times 3^{n-1}} = 1$
	$2^{n-1} \times 3^{n-1}$
٧.	
	2
	$\frac{(14)^{2} \times 4^{n+1}}{4 \times 7^n \times 16^n} = 49$
	$4 \times 7^{n} \times 16^{n}$
۸.	
	Homework
	$3^{n-2} = \frac{1}{9}$
	9
٩.	
	$\left(\frac{2}{5}\right)^{2n-1} = \frac{8}{125}$
	$\left(\frac{1}{5}\right) - \frac{1}{125}$
١٠.	
-	

	$\frac{3^{n} \times 8^{n}}{(12)^{n+1}} = \frac{1}{3}$
١١.	
Fir	and the S.S. of each of the following equations in $\mathbb R$ :
	$3^{X-3} = \left(\sqrt{3}\right)^{X+5}$
١.	
	$25 \times 3^{X-1} = 9 \times 5^{X-1}$
۲.	
	$5^{x^2-5x} = 0.0016$
٣.	
	$\frac{1}{(x+9)^4} = 0.0001$
٤.	

	$9^{X^2-1} = \frac{1}{(27)^X}$
٥.	
	If $\left(\sqrt{\frac{3}{2}}\right)^x = \frac{4}{9}$ , calculate the value of : $\left(\frac{3}{2}\right)^{x+1}$
٦.	
	Homework
	$(32)^{X-3} = 8^{2X+1}$
٧.	
٧.	
	If $\frac{49^{n} \times 25^{2 \text{ n}} \times 3^{4 \text{ n}}}{7^{-n} \times 15^{4 \text{ n}}} = 343$ , then calculate the value of: $6^{2n}$
۸.	
•	
	If $3^{X} = 27$ , $4^{X+y} = 1$ , calculate the value of each of: X and y
٩.	

# Lesson (12) Operations on integer powers



## Complete each of the following:

- The simplest form of the expression:  $2^{-3} \times 2^{-2} \div 4^{-3} = \dots$
- The simplest form of the expression:  $4^3 \times 3^{-2} \times (\sqrt[3]{-8})^{-5} = \dots$

#### Homework

- The simplest form of the expression:  $2^{-3} \times 3^{-2} \div 6^{-4} = \cdots$
- The simplest form of the expression:  $(3^{-2})^3 \div 9^{-3} \times (-2)^{-1} = \cdots$

#### Choose the correct answer:

- $\square$  The expression:  $\frac{3^{x} \times 3^{x} \times 3^{x}}{3^{x} + 3^{x} + 3^{x}}$  equals ......
- (a)  $3^{2 X-1}$
- (b)  $3^{1-2}$  x
- (c)  $3^{x^3-3}x$
- (d)  $3^3 x x^3$

- $\square$   $(5^{X+2}-5^{X+1}) \div 5^X = \dots$ 
  - (a) 5

(b) 10

(c) 15

(d) 20

#### Homework

- The value of the expression:  $3^5 + (\sqrt{3})^{10} 2(3)^5 = \cdots$
- (a) zero

(b)  $3^5$ 

- (c)  $\left(\sqrt{3}\right)^5$
- (d)  $2(3)^5$
- The simplest form of the expression :  $\sqrt{4 \times \sqrt{16} \div \sqrt[3]{8} 2^2} = \cdots$ 
  - (a) 2

٣.

(b) 4

(c)8

(d) 16

- If  $x = \sqrt{3}$ ,  $y = \sqrt{5}$ , then:  $\frac{x^8 y^8}{x^4 + y^4} = \dots$ 
  - (a) 4

(b) -4

(c) 16

(d) - 16

	$\left(\sqrt{5}\right)^5 \div 5\sqrt{5} + 2\sqrt{3} \times \sqrt{3}$
١.	
	$(\sqrt{3})^{-3} \times 3\sqrt{3} + (\sqrt{3})^{-4} \div (\sqrt{3})^{-10}$
۲.	
	$ \frac{\left(\sqrt{3}\right)^7 \times \left(\sqrt{3}\right)^{-5} - \left(\sqrt{3}\right)^2}{} $
٣.	$\frac{(\sqrt{3})^7 \times (\sqrt{3})^{-5} + (\sqrt{3})^2}{(\sqrt{3})^7 \times (\sqrt{3})^{-5} + (\sqrt{3})^2}$
	If $a = \sqrt{2}$ , $b = \sqrt{3}$ , find the numerical value of: $\frac{b^4 - a^4}{b^2 + a^2}$ «1» $\frac{a^3 + b^3}{a + b}$ «5 - $\sqrt{6}$ »
٤.	$\begin{array}{c c} \hline 1 & \frac{b-a}{b^2+a^2} & \text{ & & & & \\ \end{array}} & \begin{array}{c c}  & a+b & \\ \end{array} & \begin{array}{c c}  & & \\ \end{array} & \begin{array}{c c}  & & & \\\\ \end{array} & \begin{array}{c c}  & & & \\\\ \end{array} & \begin{array}{c c}  & & \\$

	If the volume of the right circular cone is given by the relation : $v = \frac{1}{3} \pi r^2 h$ Find the height of the cone h if the volume is : $7.7 \times 10^2 \text{ cm}^3$ .  and its diameter length is 14 cm. $\left(\pi = \frac{22}{7}\right)$
٥.	
٦.	
٧.	$(2\sqrt{5})^4 - (\sqrt{5})^{-3} \times (5\sqrt{5})^2 \div 5\sqrt{5}$

# The probability

The probability of occurrence of a certain event =  $\frac{\text{the number of times of repeating this outcome}}{\text{the number of all possible outcomes}}$ 

The expected number for occurrence of a certain event

= the probability of its occurrence  $\times$  the total number of given individuals

The random experiment is an experiment, where all its possible outcomes are known before doing it but we can't determine the actual outcome.

The sample space is the set of all possible outcomes of a random experiment and it is denoted by S.

The number of its elements is denoted by n (S)

#### The event

It is a subset of the sample space.

The probability of occurrence of an event A 

S is denoted by P (A)

It is found by using the relation:

$$P(A) = \frac{\text{the number of elements of A}}{\text{the number of elements of the sample space}} = \frac{n(A)}{n(S)}$$

#### Remarks

۲.

- The impossible event: is the event which cannot occur.
  - i.e. The probability of the impossible event equals zero.
- 2 The certain event: is the event whose outcomes are all possible.
  - **i.e.** The probability of the certain event = 1
- 3 The probability of any event is not less than zero and it is not more than 1
  - **i.e.** For any event A,  $0 \le P(A) \le 1$  **i.e.**  $P(A) \in [0, 1]$

## Complete each of the following:

For every event A, we find that  $P(A) \in \cdots$ 

10 cards are numbered from 1 to 10 A card is drawn randomly, then the probability that the card carries a prime number = ........

	box randomly, then the probability that the drawn ball is blue =	
٤.	In the experiment of throwing a fair die and observing the number on the upper factor then the probability of getting a number less than 1 equals	æ :
	A box contains 48 oranges , 4 of them are bad. If we draw an orange at random ,	
٥.	then the probability that the drawn orange is bad =	
	and the probability that it is not bad =	
_	A city has 200000 people. The probability that a person gets infected by a disease i	'n
٦.	this city is 0.003 The expected number of infection is people.	
,	A factory produces 400 lamps daily, if the probability that the lamp is	
<b>′</b> .	defective = 0.02, then the expected number of good lamps produced daily is	
	Homework	
	The probability of the impossible event =	
١.	and the probability of the certain event =	
۹.	If a fair coin is tossed once , then the probability of appearance of a head =	
١٠.	A bag contains 12 balls , 4 of them are red , 6 are green and the rest are blue. If or ball is chosen randomly , then the probability of getting a blue ball =	ne
	July 10 thousan randomly 7 aren and producting of getting a creat data	
١١.	In the experiment of throwing a fair die and observing the number on the upper fac	e
	then the probability of getting a number greater than 4 is	
١٢.	If the probability of the occurrence of an event is $\frac{5}{8}$ , then the probability of the non-occurrence of this event is	
	A room has 3 doors numbered from 1 to 3 One student goes out from one door.	
۱۳.	The probability that he goes out from the second door is	
$\boldsymbol{C}$	Choose the correct answer:	
	Which of the following may be the probability of an event?	
١.	5 J F	

J	A basket has cards labelled by the numbers from 1 to 20. If a card is selected randomly, what is the probability that the number labelled on the card is divisible by 6?					
۲.	(a) $\frac{3}{20}$	(b) $\frac{4}{20}$	(c) $\frac{5}{20}$	$(d) \frac{6}{20}$		
٣.	then the prob		-	he first player win is 0.25, the competition continues		
٤.	_			66 pupils.16 of them are that the pupil is a boy?  (d) $\frac{1}{36}$		
٥.	triangle or rig	isked to draw a triangle that – angled triangle or at the student draw a riat (b) $\frac{1}{3}$	obtuse – angled triang	-		
٦.		re white. One ball is cl		red, $\frac{1}{3}$ of them are black that the chosen ball is  (d) zero		
٧.	_	an of transport, then the		ce the probability of using worker use a mean of  (d) 2		
۸.	20 yellow bal		-	ellow. If the box contains oall randomly is $\frac{1}{4}$ , what		
	(a) 5	(b) 25	(c) 60	(d) 80		

٩.	In a mixed school, there are 1500 pupils. A random sample formed from 200 pupils is selected. It is found that the number of girls equals 90. What is the expected number of girls in the school?						
	(a) 600 girls	(b) 625 girls	(c) 650 girls	(d) 675 girls			
١٠.		poard two squares are arget, then the proba		8 cm.			
			ework				
١١.	If the probability is	that a pupil succeeds	s is 70%, then the pro	bability of his failure			
	(a) 0.7	(b) 0.07	(c) 0.3	(d) 0.03			
۱۲.	In an experiment of throwing a fair die, then the probability of appearing a number not equal to 2 in the upper face is						
	(a) $\frac{1}{6}$	(b) $\frac{2}{3}$	(c) $\frac{1}{3}$	(d) $\frac{5}{6}$			
۱۳.	If a coin is thrown 400 times, then the most expected number of appearing tail is						
	(a) 204	(b) 199	(c) 240	(d) 195			
١٤.	There are 25 boys and 20 girls in a classroom. One pupil is chosen randomly. The probability that the chosen pupil is a girl equals						
	(a) $\frac{1}{20}$	(b) $\frac{4}{9}$	(c) $\frac{1}{25}$	(d) $\frac{5}{9}$			
10.			balls and one red ball. bability that the selecte				
	(a) $\frac{1}{2}$	(b) $\frac{1}{3}$	(c) $\frac{2}{3}$	(d) $\frac{1}{6}$			
١٦.	of selecting a pu	pil whose age is less	year preparatory is 36 than or equal to 13 year ass whose ages are mo	•			
	(a) 23	(b) 24	(c) 30	(d) 32			

	In a mixed scho				boys to th	ne number of	girls is 7:9
١٧.	A pupil is selection. The probability	, ,			ıls		•
	(a) zero	(b) $\frac{7}{16}$	7	(c) $\frac{9}{16}$		(d) 7	
	The following practise a certa probability that	in game. If a	pupil is sel	ected rando			
١٨.	Game	Swimming	Handball	Athletics	Football	Gymnastics	Boxing
	Number	20	40	30	50	10	10
	(a) $\frac{1}{16}$	(b) 25	%	(c) $\frac{1}{4}$ %	6	(d) $\frac{5}{16}$	
١٩.	A spinner game is divided into two parts $X$ and $y$ , then the point is rolled 400 rounds, it stopped 98 times in the region $X$ , then which of the following figures pointer points to the region $X$ ?						
E	Ssay prob  Selecting rate Find the proba  an even num  an a number is a  prime num	andomly a candility of geto berenot divisible	<b>ting a card</b> by 10	that carries	s : nber divisi		
١.							

	Drawing randomly a coloured mar	ble out of a box containing 12 red marbles,							
	18 white marbles and 20 blue marbles. Find the probability of selecting:								
	1 a white marble.	2 a red marble.							
	3 a yellow marble.	4 a non-red marble.							
۲.	5 a red or blue marble.	<u> </u>							
	A box contains 80 similar balls. Some of	them are red and the remained are blue.							
٣.	If the probability of drawing a red ball is	If the probability of drawing a red ball is $\frac{1}{4}$ , find the number of blue balls.							
	A garment factory in the Tenth of Rai	madan City produces 6000 units							
	daily. As a sample of 1000 units was examined, 20 defective units were								
٤.	found. Calculate the number of defective units.								
	In a fruit packing plant, 30% of fruits	is not suitable for exporting							
	because the size is too small. How many tons can be exported in 10								
٥.	days if 20 tons of fruits are delivered back daily to the factory?								
	A calculator manufacturing company	examined randomly electronic							
	circuits in a sample of 200 units. The defe	ective production was 6%							
	How many units are out of order in thi	1 How many units are out of order in this sample?							
	If the total production in one month was 1500 units, how many								
٦.	units are functional units of marketing	?							

 $\square$  A survey has been conducted on 100 students about their favourite games which they practise. The result was as follows:

Favourite game	·/s	190	Si	<b>.</b> 3	<b>"</b>
	Football	Handball	Athletics	Tennis	Hockey
Number of students	44	27	12	4	13

Find the probability if a student prefer
--

- (a) Practising football.
- (b) Practising handball.
- (c) Practising athletics.

- (d) Practising tennis.
- (e) Practising hockey.
- If the number of students is 600, how many students are predicted to practise hockey?
- In producing 300 electric lamps, 18 units were found defective.
- 1 What is the probability of a unit to be a defective unit?
- 2 What is the probability of a functional unit?
- Is it possible for a unit to be a functional unit and out of order unit at the same time?
- 4 Find the sum of the probability of a defective unit and the probability of a functional unit. What do you observe?
- 5 If a daily production of this factory was 1600 electric lamps, find the number of the functional units in that day.

#### Homework

- A numbered card is selected randomly from a set of similar cards numbered from 1 to 24 Find the probability of getting a card that carries:
- 1 a multiple of 4
- 3 a multiple of 4 and 6 together
- 5 a number divisible by 25

2 a multiple of 6

.......

4 a multiple of 4 or 6

......

a positive integer less than 25

	A class has 50 stude If a student is chose		•	•		•	)
١٠.							
11.	Drawing randon coloured red, white Estimate how many	green and y	ellow, the pr	_	•	2	
	The following t	table shows t	he evaluatio	n of 50 stud	lents in one	month:	
	A student is randomly selected. What is the probability of getting a score of :						
	1 Excellent	2	Good				
١٢.	3 Failed	4	Less than go	od			
	Estimate	Excellent	Very good	Good	Pass	Fail	
	Number	6	9	11	16	8	

Estimate	Excellent	Very good	Good	Pass	Fail
Number	6	9	11	16	8



## Equality of the areas of two parallelograms

## Theorem 1

Surfaces of two parallelograms with common base and between two parallel straight lines, one is carrying this base, are equal in area.

#### Corollary 1

The parallelogram and the rectangle with common base and between two parallel straight lines are equal in area.

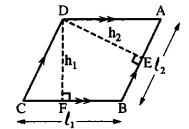
## Corollary 2

The area of the parallelogram = the length of the base  $\times$  its corresponding height.

#### Remark

#### In the opposite figure:

If ABCD is a parallelogram, DF is the corresponding height of the base  $\overline{BC}$  and DE is the corresponding height of the base  $\overline{AB}$ , then: The area of the parallelogram.



$$ABCD = BC \times DF = AB \times DE$$

$$i.e. \ l_1 \times h_1 = l_2 \times h_2$$

## Corollary 3

The parallelograms with bases equal in length and lying on a straight line, while the opposite sides to these bases are on another straight line, are equal in area.

### Corollary 4

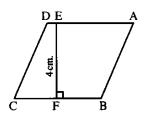
Area of a triangle is equal to half of area of a parallelogram if they have a common base lying on one of two parallel straight lines including them.

### Corollary 5

Area of the triangle =  $\frac{1}{2}$  of the length of the base × its corresponding height

## Complete each of the following:

If the area of  $\triangle 7$  ABCD = 400 cm<sup>2</sup>, then  $BC = \cdots cm$ .



If the area of  $\square$  ABCD = 600 cm<sup>2</sup>,

then 
$$CD = \cdots cm.$$

$$BE = \cdots cm.$$

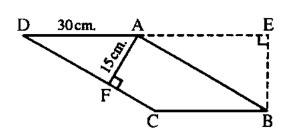
١.

۲.

٣.

٤.

٦.

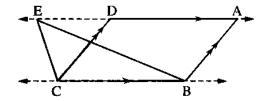


#### In the opposite figure:

ABCD is a parallelogram and  $E \in \overrightarrow{AD}$ 

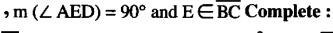
Complete the following:

- 1 The area of  $\triangle$  EBC = ..... the area of  $\triangle$  ABCD
- 2 If the area of  $\triangle$  EBC = 20 cm<sup>2</sup>, then the area of  $\triangle$  ABCD = ...... cm<sup>2</sup>

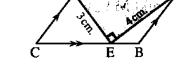


#### In the opposite figure:

ABCD is a parallelogram, AE = 4 cm., ED = 3 cm.



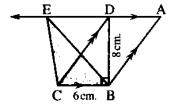




#### The area of $\triangle 7$ ABCD = ...... cm<sup>2</sup>.

#### In the opposite figure:

ABCD is a parallelogram in which, BC = 6 cm.,  $\overline{DB} \perp \overline{BC}$ , such that DB = 8 cm. and  $E \in \overline{AD}$ 



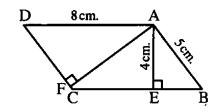
#### **Complete:**

1 The area of  $\triangle$  ABCD = ...... cm<sup>2</sup> The area of  $\triangle$  EBC = ..... cm<sup>2</sup>.

#### Homework

If ABCD is a parallelogram,

then 
$$AF = \cdots cm$$
.



Surfaces of two parallelograms with common base and between two parallel straight lines, one is carrying this base, are .........

۸.	The parallelogra		ommon base and between	two parallel straight
۹.	The area of the p	parallelogram = ······	·· × ·······	
١٠.		•	bases equal in length and se bases are on another stra	
$C_{i}$	hoose the c	correct answ	er:	
١.	If the base lengt then its area =	-	is 7 cm. and the correspon	iding height is 4 cm.,
	(a) 11 cm <sup>2</sup> .	(b) $14 \text{ cm}^2$	(c) $22 \text{ cm}^2$	(d) $28 \text{ cm}^2$
۲.	If the area of a p	<del>-</del>	n <sup>2</sup> and its height is 5 cm.,	then the length of the
	(a) 5 cm.	(b) 7 cm.	(c) 9 cm.	(d) 30 cm.
٣.	_	rallelogram in which then its greater heig	<b>AB</b> = 5 cm. <b>BC</b> = 10 tht =	cm. and its smaller
	(a) 2 cm.	(b) 4 cm.	(c) 8 cm.	(d) 10 cm.
٤.		whose area = 50 cm <sup>2</sup>	and the length of its base	equals twice the
	(a) 50 cm.	(b) 25 cm.	(c) 10 cm.	(d) 5 cm.
٥.		-	allelogram and the area of ween two parallel straight	-
	(a) 1:2	(b) 1:3	(c) 2:1	(d) 2:3
٦.	If the area of the corresponding ba		nd its height = 7 cm., then	the length of the
	(a) 15 cm.	(b) 12 cm.	(c) 8 cm. (	d) 4 cm.

		_	n which the lengths	of the sides of th	e right angle
٧.	are 6 cm. and 9 cm. (a) 54 cm.	(b) 60 cm <sup>2</sup>	(c) 27 cm <sup>2</sup> .	(d) 15 cm	2.
۸.		•	nensions are 6 cm. a cm. and the corresp		
	(a) <	(b) >	(c) =	(d) ≠	
	' I	Ho	mework		
٩.	_	arallelogram is 50 c eight of this base =	em <sup>2</sup> and its base leng	gth = 10 cm. • the	en the
	(a) 500 cm.	(b) 5 cm.	(c) 250	) cm. (	d) 100 cm.
١٠.		two adjacent sides of cm., then its area	of a parallelogram a	re 8 cm. and 10 c	em. and its
	(a) 80 cm <sup>2</sup>	(b) 50 cm <sup>2</sup>	(c) 40	cm <sup>2</sup> (	d) 18 cm <sup>2</sup>
١١.		_	e area of the paralle e straight line paral	_	as a common
	(a) equal to	(b) half	(c) twice	(d) quar	ter
	The area of the t	riangle = ····· the	base length × the c	orresponding he	ight.
۱۲.	(a) 2	(b) $\frac{1}{2}$	(c) $\frac{1}{4}$	(d) $\frac{1}{3}$	
۱۳.	If the base length then its area =	_	em. and the correspo	onding height = 3	3 cm. ,
	(a) 6 cm <sup>2</sup>	(b) 12 cm <sup>2</sup>	(c) 24 cm.	(d) 34 cm	n <sup>2</sup> .
١٤.	The triangle who height =	ose base length is 12	2 cm. and its area is	48 cm <sup>2</sup> , the corr	esponding
	(a) 3 cm.	(b) 4 cm.	(c) 6 cm.	(d) 8 cm.	
	If ABCD is a pa	rallelogram with are	ea 100 cm <sup>2</sup> and E∈	AD,	
10.	then the area of	Δ EBC =			_
	(a) 25 cm <sup>2</sup>	(b) $50 \text{ cm}^2$	(c) $100 \text{ cm}^2$	(d) 200 c	m <sup>2</sup> .

## Essay problems:

	In the opposite figure :	10
	ABCD is a parallelogram in which m ( $\angle$ ABC) = 150°,	D 12cm. A
	AD = 12  cm.	150° SOTE
	$AB = 8 \text{ cm.}$ $E \in \overrightarrow{CB} \text{ and } \overrightarrow{AE} \perp \overrightarrow{CB}$	B E
	Find: The area of ABCD	« 48 cm <sup>2</sup> . »
١.		
·		
	In the opposite figure :	
		F D E A
	ABCD and EBCF are two parallelograms,	
	ABCD and EBCF are two parallelograms, $E \in \overrightarrow{AD} \text{ and } F \in \overrightarrow{AD}$	
	$E \in \overrightarrow{AD}$ and $F \in \overrightarrow{AD}$	C B
	$E \in \overrightarrow{AD}$ and $F \in \overrightarrow{AD}$	
	$E \in \overrightarrow{AD}$ and $F \in \overrightarrow{AD}$	
۲.	$E \in \overrightarrow{AD}$ and $F \in \overrightarrow{AD}$	
۲.	$E \in \overrightarrow{AD}$ and $F \in \overrightarrow{AD}$	
۲.	$E \in \overrightarrow{AD}$ and $F \in \overrightarrow{AD}$	
۲.	$E \in \overrightarrow{AD}$ and $F \in \overrightarrow{AD}$	
۲.	$E \in \overrightarrow{AD}$ and $F \in \overrightarrow{AD}$	
۲.	$E \in \overrightarrow{AD}$ and $F \in \overrightarrow{AD}$	
۲.	$E \in \overrightarrow{AD}$ and $F \in \overrightarrow{AD}$	

	In the opposite figure :
	ABCD is a rectangle, $\overline{AE} /\!/ \overline{DF}$
	Prove that :
	The area of the figure ABCM = the area of the figure DMEF $\frac{1}{E}$ $\frac{1}{E}$ $\frac{1}{E}$ $\frac{1}{E}$ $\frac{1}{E}$ $\frac{1}{E}$
٣.	
	In the opposite figure :
	ABC is a right-angled triangle at A,
	$\overline{AD} \perp \overline{BC}$ , $AB = 4$ cm. and $AC = 3$ cm.
	Find: 11 The area of $\triangle$ ABC 2 The length of $\overline{AD}$ B 4cm. A
٤.	

	☐ In the opposite figure :
	ABCD is a rectangle and $E \in \overrightarrow{BC}$ Prove that: The area of $\triangle$ DAE = the area of $\triangle$ ABC
٥.	
	In the opposite figure:  ABCD and ABMN are two parallelograms
	and $M \in \overline{CD}$ Prove that:  The area of $\triangle$ EBC = $\frac{1}{2}$ the area of $\square$ ABMN
4	
٦.	

In the opposite figure :	E D	
ABCD is a parallelogram, $E \subseteq \overline{AD}$ and $\overline{BE} \cap \overline{CD} = \{F\}$		
<b>Prove that :</b> The area of $\triangle$ AFD = the area of $\triangle$ EFC	C	<b>\</b> E
		• • • •
		• • • •
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		• • • •
	• • • • • • • • • • • • • • • • • • • •	• • • •
In the opposite figure:	Y X B	
$\overrightarrow{AB}$ // $\overrightarrow{DE}$ , X and Y $\in \overrightarrow{AB}$	*	n z
$\overrightarrow{AB}$ // $\overrightarrow{DE}$ , X and Y $\in \overrightarrow{AB}$ , XDEY is a rectangle and $\overrightarrow{AD}$ // $\overrightarrow{BE}$	<b>₽</b> →	d L
AB // DE , X and Y ∈ AB , XDEY is a rectangle and AD // BE  1 Find the area of the figure ABED	75 E	n E
AB // DE , X and Y ∈ AB , XDEY is a rectangle and AD // BE  1 Find the area of the figure ABED 2 If : AD = 30 cm. , find the length of the perpendicular	E 12cm. D	
AB // DE , X and Y ∈ AB , XDEY is a rectangle and AD // BE  1 Find the area of the figure ABED	75 E	
AB // DE , X and Y ∈ AB , XDEY is a rectangle and AD // BE  1 Find the area of the figure ABED 2 If : AD = 30 cm. , find the length of the perpendicular	E 12cm. D	
AB // DE , X and Y ∈ AB , XDEY is a rectangle and AD // BE  1 Find the area of the figure ABED 2 If : AD = 30 cm. , find the length of the perpendicular	E 12cm. D	
AB // DE , X and Y ∈ AB , XDEY is a rectangle and AD // BE  1 Find the area of the figure ABED 2 If : AD = 30 cm. , find the length of the perpendicular	E 12cm. D	
AB // DE , X and Y ∈ AB , XDEY is a rectangle and AD // BE  1 Find the area of the figure ABED 2 If : AD = 30 cm. , find the length of the perpendicular	E 12cm. D	
AB // DE , X and Y ∈ AB , XDEY is a rectangle and AD // BE  1 Find the area of the figure ABED 2 If : AD = 30 cm. , find the length of the perpendicular	E 12cm. D	
AB // DE , X and Y ∈ AB , XDEY is a rectangle and AD // BE  1 Find the area of the figure ABED 2 If : AD = 30 cm. , find the length of the perpendicular	E 12cm. D	
AB // DE , X and Y ∈ AB , XDEY is a rectangle and AD // BE  1 Find the area of the figure ABED 2 If : AD = 30 cm. , find the length of the perpendicular	E 12cm. D	
AB // DE , X and Y ∈ AB , XDEY is a rectangle and AD // BE  1 Find the area of the figure ABED 2 If : AD = 30 cm. , find the length of the perpendicular	E 12cm. D	

......

	In the opposite figure :						
	ABC is a triangle in which BC = $6.5 \text{ cm}$ .						
	$AC = 6 \text{ cm.}$ $AE \perp \overline{BC}$ $\overline{BD} \perp \overline{AC}$ and $BD = 5 \text{ cm.}$						
	Find: 1 The area of $\triangle$ ABC	The length of $\overline{AE}$	В				
			•••				
			•••				
٩.			•••				
١.			•••				
			•••				
			• • • •				
			•••				
			• • • •				
			•••				
			••••				
	In the opposite figure :	Ę					
	ABCD is a rectangle, ABEF is a par	rallelogram					
	$,D \in \overline{CF}, X \in \overline{BE}, E \in \overline{CF}$	Е					
	AB = 4 cm. and BC = 10 cm.	D	A				
	Find by proof:		4cm				
	1 The area of ∠ ABEF	C 10cm.	<u>ا</u> 8				
	The area of Δ XAF	« 40 cm <sup>2</sup> • 20 cr	m. »				
			•••				
١٠.							
			•••				
			• • • •				
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			•••				

	In the opposite figure : ABCD and EBCF are two parallelograms , $\overline{BE} \cap \overline{CD} = \{L\}$ , $D \in \overline{AF}$ and $E \in \overline{AF}$ Prove that : 1 The area of $\triangle$ ABL = the area of $\triangle$ FCL
	The area of the figure ABCL = the area of the figure FCBL
١١.	
•	

D

## Equality of the areas of two triangles



Two triangles which have the same base and the vertices opposite to this base on a straight line parallel to the base have the same area.

#### Corollary 1

Triangles of bases equal in length and lying between two parallel straight lines are equal in area.

#### Corollary 2

The median of a triangle divides its surface into two triangular surfaces equal in area.

#### Corollary 3

Triangles with congruent bases on one straight line and have a common vertex are equal in areas.

#### Theorem 3

If two triangles are equal in area and drawn on the same base and on one side of it, then their vertices lie on a straight line parallel to this base.

## Complete each of the following:

- If ABC is a triangle, D is the midpoint of  $\overline{BC}$ , then: The area of  $\triangle ABD$  = the area of  $\triangle \cdots$
- If  $\overline{XL}$  is a median in  $\Delta XYZ$ , then the area of  $\Delta XYZ = \cdots$  the area of  $\Delta XYL$
- The triangle XYZ in which  $L \subseteq \overline{YZ}$  such that  $YL = \frac{1}{2} LZ$ , then: The area of  $\Delta XYL = \cdots$  the area of  $\Delta XYZ$

#### Homework

- The two triangles drawn on a common base and their vertices located on a straight line parallel to the base are ........
- o. Triangles with congruent bases and drawn between two parallel lines are .........
- The median in the triangle divides its area into ........

## Essay problems:

	In the opposite figure :
	D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$
	Prove that: The area of $\triangle$ BDE equals the area of $\triangle$ CDE
	C B
١.	
	In the opposite figure :
	<u> </u>
	ABCD is a quadrilateral in which $\overrightarrow{AD} / / \overrightarrow{BC}$ and $\overrightarrow{BA} \cap \overrightarrow{CD} = \{E\}$
	E
	ABCD is a quadrilateral in which $\overrightarrow{AD}$ // $\overrightarrow{BC}$ and $\overrightarrow{BA} \cap \overrightarrow{CD} = \{E\}$ such that $BA = AE$
	ABCD is a quadrilateral in which $\overrightarrow{AD} / / \overrightarrow{BC}$ and $\overrightarrow{BA} \cap \overrightarrow{CD} = \{E\}$
	ABCD is a quadrilateral in which $\overrightarrow{AD}$ // $\overrightarrow{BC}$ and $\overrightarrow{BA} \cap \overrightarrow{CD} = \{E\}$ such that $BA = AE$
	ABCD is a quadrilateral in which $\overrightarrow{AD}$ // $\overrightarrow{BC}$ and $\overrightarrow{BA} \cap \overrightarrow{CD} = \{E\}$ such that $BA = AE$
۲.	ABCD is a quadrilateral in which $\overrightarrow{AD}$ // $\overrightarrow{BC}$ and $\overrightarrow{BA} \cap \overrightarrow{CD} = \{E\}$ such that $BA = AE$
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۲.	ABCD is a quadrilateral in which $\overrightarrow{AD}$ // $\overrightarrow{BC}$ and $\overrightarrow{BA} \cap \overrightarrow{CD} = \{E\}$ such that $BA = AE$

	$\overline{AC}$ // $\overline{XY}$ and F is the midpoint of $\overline{XY}$
l	<b>Prove that :</b> The area of $\triangle$ ABF = the area of $\triangle$ CBF
l	
l	
l	
l	
l	
Ļ	
	ABCD is a parallelogram. $E \in \overrightarrow{CB}$ where $BC = BE$
	ABCD is a parallelogram. $E \in \overrightarrow{CB}$ where $BC = BE$
	ABCD is a parallelogram. $E \in \overrightarrow{CB}$ where $BC = BE$
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	ABCD is a parallelogram. $E \in \overrightarrow{CB}$ where $BC = BE$
	ABCD is a parallelogram. $E \in \overrightarrow{CB}$ where $BC = BE$

	In the opposite figure :
	ABCD is a quadrilateral whose diagonals intersect at M,
	$\overline{AD}$ // $\overline{BC}$ and E is the midpoint of $\overline{AB}$ ,
	N is the midpoint of MC
	<b>Prove that :</b> The area of $\triangle$ AEM = the area of $\triangle$ DNC
٥.	
	In the opposite figure :
	In the opposite figure : $\overrightarrow{AD} / \overrightarrow{BC}, E \in \overrightarrow{BC} \text{ and } \overrightarrow{AC} / \overrightarrow{DE},$
	$\overrightarrow{AD} / \overrightarrow{BC}, E \in \overrightarrow{BC} \text{ and } \overrightarrow{AC} / \overrightarrow{DE},$
	$\overrightarrow{AD}$ // $\overrightarrow{BC}$ , $E \in \overrightarrow{BC}$ and $\overrightarrow{AC}$ // $\overrightarrow{DE}$ , $\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$ Prove that:  1 The area of $\triangle$ ABM = the area of $\triangle$ DCM = the area of $\triangle$ EMC
	$\overrightarrow{AD} / \overrightarrow{BC}$ , $E \in \overrightarrow{BC}$ and $\overrightarrow{AC} / \overrightarrow{DE}$ , $\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$ Prove that:
	$\overrightarrow{AD}$ // $\overrightarrow{BC}$ , $E \in \overrightarrow{BC}$ and $\overrightarrow{AC}$ // $\overrightarrow{DE}$ , $\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$ Prove that:  1 The area of $\triangle$ ABM = the area of $\triangle$ DCM = the area of $\triangle$ EMC
٦.	$\overrightarrow{AD}$ // $\overrightarrow{BC}$ , $E \in \overrightarrow{BC}$ and $\overrightarrow{AC}$ // $\overrightarrow{DE}$ , $\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$ Prove that:  1 The area of $\triangle$ ABM = the area of $\triangle$ DCM = the area of $\triangle$ EMC
٦.	$\overrightarrow{AD}$ // $\overrightarrow{BC}$ , $E \in \overrightarrow{BC}$ and $\overrightarrow{AC}$ // $\overrightarrow{DE}$ , $\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$ Prove that:  1 The area of $\triangle$ ABM = the area of $\triangle$ DCM = the area of $\triangle$ EMC
٦.	$\overrightarrow{AD}$ // $\overrightarrow{BC}$ , $E \in \overrightarrow{BC}$ and $\overrightarrow{AC}$ // $\overrightarrow{DE}$ , $\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$ Prove that:  1 The area of $\triangle$ ABM = the area of $\triangle$ DCM = the area of $\triangle$ EMC
٦.	$\overrightarrow{AD}$ // $\overrightarrow{BC}$ , $E \in \overrightarrow{BC}$ and $\overrightarrow{AC}$ // $\overrightarrow{DE}$ , $\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$ Prove that:  1 The area of $\triangle$ ABM = the area of $\triangle$ DCM = the area of $\triangle$ EMC
٦.	$\overrightarrow{AD}$ // $\overrightarrow{BC}$ , $E \in \overrightarrow{BC}$ and $\overrightarrow{AC}$ // $\overrightarrow{DE}$ , $\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$ Prove that:  1 The area of $\triangle$ ABM = the area of $\triangle$ DCM = the area of $\triangle$ EMC
٦.	$\overrightarrow{AD}$ // $\overrightarrow{BC}$ , $E \in \overrightarrow{BC}$ and $\overrightarrow{AC}$ // $\overrightarrow{DE}$ , $\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$ Prove that:  1 The area of $\triangle$ ABM = the area of $\triangle$ DCM = the area of $\triangle$ EMC
٦.	$\overrightarrow{AD}$ // $\overrightarrow{BC}$ , $E \in \overrightarrow{BC}$ and $\overrightarrow{AC}$ // $\overrightarrow{DE}$ , $\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$ Prove that:  1 The area of $\triangle$ ABM = the area of $\triangle$ DCM = the area of $\triangle$ EMC
٦.	$\overrightarrow{AD}$ // $\overrightarrow{BC}$ , $E \in \overrightarrow{BC}$ and $\overrightarrow{AC}$ // $\overrightarrow{DE}$ , $\overrightarrow{AC} \cap \overrightarrow{BD} = \{M\}$ Prove that:  1 The area of $\triangle$ ABM = the area of $\triangle$ DCM = the area of $\triangle$ EMC

	In the opposite figure:
	ABCD is a quadrilateral, its diagonals intersect at M
	and the area of $\triangle$ ABM = the area of $\triangle$ DCM
	Prove that: $\overline{AD} /\!/ \overline{BC}$
٧.	
٠.	
	☐ In the opposite figure :
	ABCD is a quadrilateral whose diagonals are intersecting at M
	ABCD is a quadrilateral whose diagonals are intersecting at M
	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$
	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME
	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME
	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME
	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME
۸.	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME
۸.	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME
۸.	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME
۸.	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME
۸.	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME
۸.	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME
۸.	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME
۸.	ABCD is a quadrilateral whose diagonals are intersecting at M and $E \in \overline{BM}$ where $ME = MD$ The area of $\Delta$ AMB = the area of $\Delta$ CME

### Homework

	In the opposite figure :
	$\overline{AD} / / \overline{BC}, \overline{AC} \cap \overline{BD} = \{M\},$
	E is the midpoint of $\overline{BC}$
	Prove that: The area of the figure ABEM = the area of the figure DMEC
٩.	
٠.	
	In the opposite figure :
	$\overline{AD} // \overline{BC}$ and $\overline{AC} \cap \overline{BD} = \{M\}$ ,
	D is the midpoint of $\overline{EC}$
	Prove that:
	The area of $\triangle$ MDE = the area of $\triangle$ AMB
١٠.	

	In the opposite figure :
	ABCD is a parallelogram. Its diagonals intersect at M
	in which $\overline{AD}$ // $\overline{BC}$ and B is the midpoint of $\overline{EC}$
	Prove that : The area of $\triangle$ EBD = the area of $\triangle$ ACD
`	
١.	
	In the opposite figure :
	APC is a triangle in which $D \subset \overline{AP}$ and $E \subset \overline{AC}$
	ABC is a triangle in which $D \in \overline{AB}$ and $E \in \overline{AC}$
	such that the area of $\triangle$ ABE = the area of $\triangle$ ACD
	such that the area of $\triangle$ ABE = the area of $\triangle$ ACD
	such that the area of $\triangle$ ABE = the area of $\triangle$ ACD
	such that the area of $\triangle$ ABE = the area of $\triangle$ ACD
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۲.	such that the area of $\triangle$ ABE = the area of $\triangle$ ACD

	In the opposite figure :
	ABCD is a quadrilateral in which $\overline{AD} // \overline{BC}$
	$ \mathbf{F} \in \overrightarrow{BC} \text{ and } \overrightarrow{AC} \cap \overrightarrow{BD} = \{M\} $
	The area of $\triangle$ ABM = the area of $\triangle$ ECM
	Prove that: DE // AC
۱۳.	
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# Lesson (10) Areas of some geometric figures

The area of the rhombus =  $L \times h$  where L is the side length and h is the height.

The area of the rhombus =  $\frac{1}{2}$  of the product of the lengths of its two diagonals.

If the two legs of the trapezium are equal in length, then it is called an isosceles trapezium. The following are the properties of the isosceles trapezium:

The two base angles of the isosceles trapezium are equal in measure.

The two diagonals of the isosceles trapezium are equal in length.

The isosceles trapezium has only one axis of symmetry which is the perpendicular bisector of its bases.

The area of the trapezium = half of the sum of lengths of the two parallel bases  $\times$  height

The area of the trapezium = the length of the middle base  $\times$  height

# Complete each of the following:

١.	The area of rhombus whose perimeter is 20 cm. and height 4 cm. =
۲.	The length of the diagonal of a square of area 50 cm <sup>2</sup> equals cm.
٣.	The length of side of a square whose area equals the area of a rectangle with dimensions 9 cm., 16 cm. =
٤.	The length of the middle base of a trapezium whose area = 30 cm <sup>2</sup> and height 5 cm. equals
	Homework
٥.	The area of the rhombus = the side length $\times \dots = \frac{1}{2}$ of the product of $\dots$
٦.	The area of the square = the square of the length of $\frac{2}{1}$
٧.	The length of the middle base of the trapezium equals
۸.	The area of the trapezium = half of the sum of lengths of the two parallel bases ×
٩.	The base angles of the isosceles trapezium are

١٠.	The diagonals of a	ın isosceles trapeziu	m are ·······			
Ci	Choose the correct answer:					
•	If the area of a squ	are is 50 cm <sup>2</sup> , then t	the length of its diagona	1 =		
١.	(a) 25 cm.	(b) 5 cm.	(c) 10 cm.	(d) 20 cm.		
۲.	If the perimeter of	a rhombus is 24 cm.	and its area = $30 \text{ cm.}^2 \text{ t}$	hen its height = ······		
١.	(a) 4 cm.	(b) 5 cm.	(c) 6 cm.	(d) 12 cm.		
٣.	If the product of the 6 cm., then its side	•	gonals of a rhombus = 9	6 cm <sup>2</sup> and its height is		
	(a) 12 cm.	(b) 8 cm.	(c) 6 cm.	(d) 4 cm.		
٤.	If the area of a trapezium is 32 cm <sup>2</sup> and its height is 4 cm., then the length of its middle base =					
	(a) 4 cm.	(b) 8 cm.	(c) 14 cm.	(d) 16 cm.		
The trapezium in which the length of one of its parallel bases is 15 cm., is 108 cm <sup>2</sup> and its height is 8 cm., then the length of the other base is						
	(a) 15 cm.	(b) 4 cm.	(c) 12 cm.	(d) 27 cm.		
The trapezium whose middle base length is $x$ cm. and its height = $\frac{1}{2}$ the length the middle base, its area = cm <sup>2</sup> .			$ht = \frac{1}{2} \text{ the length of}$			
	(a) $X^2$	(b) $\frac{x^2}{2}$	(c) $\frac{\chi^2}{4}$	$(\mathbf{d}) \frac{\mathbf{x}^2}{8}$		
	1	Home	work			
٧.	The area of rhomb length of the other		gth of one of its diagon	als is 5 cm., then the		
	(a) 8 cm.	(b) 4 cm.	(c) 10 cm.	(d) 15 cm.		
	The area of the squ	are whose side leng	th is 6 cm the ar	rea of the square		
۸.	whose diagonal lea	ngth is 8 cm.				
	(a) >	(b) <	(c) =	(d) <b>≡</b>		

	(a) 26 cm.	(b) 15 cm.	(c) 13 cm.	(d) 11 cm.
١٠.		e trapezium is 450 c m. , then its height	_	f its two parallel bases are
	(a) 12.5 cm.	(b) 25 cm.	(c) 36 cm.	(d) 52 cm.
F	ind the ar	rea of the f	following fig	gures:
		de length 6 cm. and		« 30 cm <sup>2</sup> »
<b>'</b> .	A rhombus who	se diagonal lengths	are 24 cm. and 10 cm	« 120 cm <sup>2</sup> »
	<b>A</b>	31	10	50 <sup>2</sup>
•	A square whose	diagonal length = 1		« 50 cm.
	A trapezium who	ose bases lengths ar	e 8 cm. and 10 cm. and	d its height = 5 cm. « 45 cm <sup>2</sup>
	A trapezium who	ose bases lengths ar	e 8 cm. and 10 cm. and gth is 7 cm. and its he	d its height = 5 cm. « 45 cm <sup>2</sup>
	A trapezium who	ose bases lengths are ose middle base length.  Ho ose side length 12 o	e 8 cm. and 10 cm. and	d its height = 5 cm. « 45 cm <sup>2</sup> ; eight = 6 cm. « 42 cm <sup>2</sup> ;
	A trapezium who	ose bases lengths are ose middle base length	e 8 cm. and 10 cm. and gth is 7 cm. and its he cm. and its height = 8 cm. and 10 cm.	d its height = 5 cm. « 45 cm <sup>2</sup> ; eight = 6 cm. « 42 cm <sup>2</sup> ; 8 cm. « 96 cm <sup>2</sup> ;
	A trapezium who	ose bases lengths are ose middle base length	e 8 cm. and 10 cm. and gth is 7 cm. and its he mework cm. and its height = 8 cm. and 10 cm.	% 96 cm. « 96 cm. »

١٠.	A trapezium whose middle base length is 12 cm. and its height = 8 cm. « 96 cm <sup>2</sup> .»
E	ssay problems:
	A square whose area equals the area of the rectangle whose dimensions are 2 cm. and 9 cm.  Find the length of its diagonal.  «6 cm.»
١.	
۲.	Two pieces of land have equal areas, one of them has the shape of a rhombus whose diagonals are 18 m. and 24 m., and the other one has the shape of a trapezium whose height is 12 m. Find the length of its middle base.  « 18 m. »
١.	
	The area of a trapezium is 180 cm <sup>2</sup> and its height is 12 cm. Find the lengths of its parallel bases if the ratio between their lengths is 3:2 «18 cm., 12 cm.»
<b>~</b> .	
	Homework
	Two land pieces are equal in area, the first is in the shape of a square and the second is in the shape of a rhombus whose diagonals lengths are 8 metres and 16 metres.
٤.	Find the perimeter of the square-shaped piece. « 32 cm. »



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The figure		The perimeter	The area
The triangle	h	The sum of the lengths of its three sides	$\frac{1}{2}$ of the base length × height $= \frac{1}{2} \ell \times h$
The parallelogram		The sum of lengths of two adjacent sides $\times$ 2 = 2 ( $\ell_1 + \ell_2$ )	The base length $\times$ height $= \ell_1 \times \mathbf{h}_1 = \ell_2 \times \mathbf{h}_2$
The rectangle		2 (Length + Width) $= 2 (l + w)$	Length $\times$ Width $= \ell \times W$
The square		Side length $\times 4 = 4 \ell$	Square of side length = $\ell^2$ or $\frac{1}{2}$ of the square of its diagonal length = $\frac{1}{2}$ r <sup>2</sup>
The rhombus	T <sub>2</sub>	Side length $\times 4 = 4 \ell$	Side length × height = $\ell$ × h or $\frac{1}{2}$ the product of the lengths of the two diagonals = $\frac{1}{2}$ r <sub>1</sub> × r <sub>2</sub>
The trapezium		The sum of lengths of its sides	$\frac{1}{2}$ the sum of lengths of the two parallel bases × height $= \frac{1}{2} (\ell_1 + \ell_2) \times h$ or the length of the middle base × height $= \ell \times h$



# Lesson (1/) Similarity

It is said that the two polygons  $P_1$  and  $P_2$  (of the same number of sides) are similar if the following two conditions are verified together:

- Their corresponding angles are equal in measure.
- The corresponding side lengths are proportional.

  In this case, we write the polygon  $P_1 \sim$  the polygon  $P_2$ That means the polygon  $P_1$  is similar to the polygon  $P_2$

#### Remark (1)

In the two similar polygons  $P_1$  and  $P_2$ , the constant ratio among the lengths of the corresponding sides of  $P_1$  and  $P_2$  is called the ratio of enlargement or the drawing scale.

#### If the constant ratio is:

- Greater than 1, then the polygon P<sub>1</sub> is an enlargement to the polgyon P<sub>2</sub>
- Less than 1 , then the polygon P<sub>1</sub> is a minimizing of the polgyon P<sub>2</sub>
- Equal to 1, then the polygon P<sub>1</sub> is congruent to the polgyon P<sub>2</sub>

### Remark (2)

In order that two polygons are similar, the two conditions should be verified together and verifying one of them only is not enough to be similar.

### Remark (3)

The congruent polygons are similar but it is not necessary that the similar polygons are congruent.

### Remark (4)

All regular polygons of the same number of sides are similar.

#### Remark (5)

If each of two polygons is similar to a third polygon, then they are similar.

#### Remark (6)

The order of corresponding vertices should be kept in giving names of similar polygons that to help us finding the proportional sides lengths and the equal angles in measures.

ı	•	O
ı	•	C.

The ratio between the perimeters of two similar polygons = the ratio between the lengths of two corresponding sides.

#### → A geometric fact : 🕶

The two triangles are similar if one of the two following conditions is verified:

- The measures of their corresponding angles are equal.
- 2 The lengths of their corresponding sides are proportional.

#### Remarks

- 1 The two right-angled triangles are similar if the measure of an acute angle in one of them is equal to the measure of an acute angle in the other.
- 2 The two equilateral triangles are similar.
- The two isosceles triangles are similar if the measure of an angle in one of them equals the measure of the corresponding angle in the other.

# Complete each of the following:

١.	If the measures of the corresponding angles in the two triangles are equal, then the two triangles are
۲.	If we have two polygons, their corresponding angles are
٣.	If the ratio between the lengths of two corresponding sides in two similar triangles is equal to 1, then the two triangles are
٤.	If two polygons are similar and the ratio between the lengths of two corresponding sides is 3:4, then the ratio between their perimeters is  Homework
٥.	If two polygons are similar, then the corresponding are equal in measure.
٦.	If two polygons are similar, then the corresponding are proportional.
٧.	If each of two polygons is similar to a third, then they are
۸.	The two triangles are similar if the corresponding are proportional.

# Choose the correct answer:

If the ratio between the lengths of two corresponding sides of two squares is 1 and the perimeter of one of them is 20 cm., then the area of the other square = .........

(a)  $20 \text{ cm}^2$ 

١.

۲.

٣.

٤.

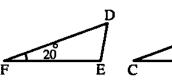
٥.

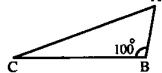
- (b)  $25 \text{ cm}^2$
- (c)  $16 \text{ cm}^2$
- (d) 25 cm.

## In the opposite figure:

If  $\triangle$  ABC  $\sim$   $\triangle$  DEF, then m ( $\angle$  A) = ......

- (a) 20°
- $(b) 60^{\circ}$
- (c) 80°
- (d) 100°





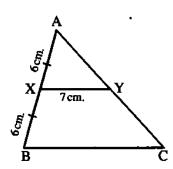
## In the opposite figure:

If  $\triangle$  ABC  $\sim$   $\triangle$  AXY,

AX = XB = 6 cm.

XY = 7 cm., then  $BC = \cdots$ 

- (a) 6 cm.
- (b) 7 cm.
- (c) 12 cm.
- (d) 14 cm.



## Homework

If  $\triangle$  ABC  $\sim$   $\triangle$  DEF and AB =  $\frac{1}{5}$  DE, then perimeter of  $\triangle$  ABC = ..... perimeter of  $\triangle$  DEF

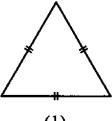
(a) 5

(b) 1

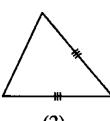
(c)  $\frac{1}{5}$ 

(d)  $\frac{2}{5}$ 

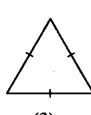
In the following figures, there are two similar triangles, they are .........



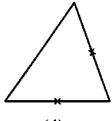
(1)



(2)



(3)



(4)

- (a) 1,2
- (b) 1,3

- (c) 1,4
- (d) 2,4

In the following figures, there are two similar triangles, they are ......... ٦. (1) (4) (3) (b) 1,3(a) 1, 2(c) 2,4(d) 1,4In the opposite figure: If  $\triangle$  ABC  $\sim$   $\triangle$  DEF, then EF = ....... (a) 5 cm. (b) 6 cm. (c) 8 cm. (d) 10 cm. Essay problems: In the opposite figure:  $\Delta$  ABC  $\sim$   $\Delta$  XYZ Find: AC and XY « 14 cm. , 3 cm. » ١.

In the opposite figure:		5cm Vch
<b>1 Prove that :</b> $\triangle$ ABC and $\triangle$ XYZ	Z are similar.	C 7cm. E
If: $m (\angle B) + m (\angle C) = 60^{\circ}$ ,		10cm. X
find: $m (\angle X)$	« 120°»	Z 14cm.
In the emperite forms	,	
In the opposite figure:		
If $\triangle AXY \sim \triangle ABC$		y 5cm.
If $\triangle$ AXY $\sim$ $\triangle$ ABC XY = 5 cm. and BC = 10 cm.	V is the midualist of $\overline{AC}$	y 5cm.
If $\triangle$ AXY $\sim$ $\triangle$ ABC XY = 5 cm. and BC = 10 cm.	Y is the midpoint of $\overline{AC}$	Y 5cm.
If $\triangle$ AXY $\sim$ $\triangle$ ABC XY = 5 cm. and BC = 10 cm.	Y is the midpoint of $\overline{AC}$	
If $\triangle$ AXY $\sim$ $\triangle$ ABC XY = 5 cm. and BC = 10 cm.	Y is the midpoint of AC	
If $\triangle$ AXY $\sim$ $\triangle$ ABC XY = 5 cm. and BC = 10 cm.	Y is the midpoint of AC	
If $\triangle$ AXY $\sim$ $\triangle$ ABC XY = 5 cm. and BC = 10 cm.	Y is the midpoint of AC	
If $\triangle$ AXY $\sim$ $\triangle$ ABC XY = 5 cm. and BC = 10 cm.	Y is the midpoint of AC	
If $\triangle$ AXY $\sim$ $\triangle$ ABC XY = 5 cm. and BC = 10 cm.	Y is the midpoint of AC	
If $\triangle$ AXY $\sim$ $\triangle$ ABC XY = 5 cm. and BC = 10 cm.	Y is the midpoint of AC	
If $\triangle$ AXY $\sim$ $\triangle$ ABC XY = 5 cm. and BC = 10 cm.	Y is the midpoint of AC	

	In the opposite figure:  The polygon ABCD ~ the polygon XYZL  D  D  C  D  D  D  D  D  D  D  D  D  D
	If AB = 10.5 cm., BC = 8 cm., CD = 5 cm.,
	DA = 6 cm. and LX = 3 cm. $\frac{7}{X}$ ? Y A 10.5 cm. B
	Find the length of each of: $\overline{XY}$ , $\overline{YZ}$ and $\overline{ZL}$ «5.25 cm., 4 cm., 2.5 cm.)
٤.	
	In the opposite figure :
	$\triangle ABC, D \in \overline{AB}, E \in \overline{AC}$
	AE = 4  cm. $EC = 5  cm.$ $BC = 7.5  cm.$
	$AD = 3 \text{ cm.}$ $m (\angle AED) = m (\angle B)$ and $m (\angle C) = 93^{\circ}$
	1 Prove that : $\triangle AED \sim \triangle ABC$
	2 Find the length of each of: BD and m (∠ ADE) «9 cm. ,93° »
٥.	

	In the opposite figure :	n
	ABC is a right-angled triangle at B in which:	<b>^</b> ,
	AB = 3 cm., BC = 4 cm. and $\overline{BD} \perp \overline{AC}$	\\Zighta_{\text{th}}
	1 Prove that : $\triangle$ BAC $\sim$ $\triangle$ DAB	D A
	<b>2</b> Find the length of each of : $\overline{AD}$ and $\overline{DC}$ « 1.8	cm. , 3.2 cm. »
٦.		
٠.		
		•••••
	Two similar triangles, one of them has a perimeter of 74 cm. and the si of the other are 4.5 cm., 6 cm. and 8 cm.  Find the length of the longest side in the first triangle.	des lengths « 32 cm. »
	of the other are 4.5 cm., 6 cm. and 8 cm.	
	of the other are 4.5 cm., 6 cm. and 8 cm.	
	of the other are 4.5 cm., 6 cm. and 8 cm.	
	of the other are 4.5 cm., 6 cm. and 8 cm.	
٧.	of the other are 4.5 cm., 6 cm. and 8 cm.	
٧.	of the other are 4.5 cm., 6 cm. and 8 cm.	
٧.	of the other are 4.5 cm., 6 cm. and 8 cm.	
٧.	of the other are 4.5 cm., 6 cm. and 8 cm.	
٧.	of the other are 4.5 cm., 6 cm. and 8 cm.	
٧.	of the other are 4.5 cm., 6 cm. and 8 cm.	
٧.	of the other are 4.5 cm., 6 cm. and 8 cm.	
٧.	of the other are 4.5 cm., 6 cm. and 8 cm.	

# Homework

	Using the shown data in the figure, then prove that:				
	$\Delta$ XYZ and $\Delta$ BCA		Z 3cm P		
	are similar, then find	d			
	the perimeter of $\Delta$ XYZ	« 9 cm. »	X C 16cm. B		
۸.					
	In the opposite figure :				
	$\overline{AC} / \overline{ED}, \overline{AD} \cap \overline{CE} = \{B\}$		C 5cm. A		
	AC = 5  cm $BE = 8  cm$ $AB = 3  cm$ and $BD = 6  cm$ .				
	1 Prove that : $\triangle$ ABC $\sim$ $\triangle$ DBI		E Sch		
	Find the length of each of: 1				
	Find: the ratio of enlargement		D E « 4 cm. , 10 cm. , 2 »		
	_				
٩.					
•					
		•••••			

	☐ In the opposite figure :
	m ( $\angle$ AED) = m ( $\angle$ B), AD = 3 cm.
	AE = 4.5  cm. and $BD = 6  cm.$
	1 Prove that : $\triangle$ ADE $\sim$ $\triangle$ ACB
	2 Find the length of : EC «1.5 cm. »
• .	
•	



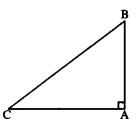
# The converse of Pythagoras' theorem

'We studied Pythagoras' theorem last year.

In the following, we will remind you of what you have studied.

If ABC is a right-angled triangle at A, then  $(BC)^2 = (AB)^2 + (AC)^2$ 

Now we shall study the converse of Pythagoras' theorem.

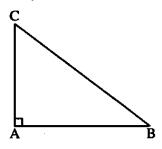


In a triangle, if the sum of the areas of two squares on two sides is equal to the area of the square on the third side, then the angle opposite to this side is a right angle.

In  $\triangle$  ABC , if:

$$(AB)^2 + (AC)^2 = (BC)^2$$
,

then:  $m (\angle A) = 90^{\circ}$ 



→ We can state this theorem as follows : →

In a triangle, if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides, then the angle opposite to this side is a right angle.

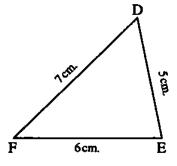
## Corollary

In  $\triangle$  ABC, if  $\overline{AC}$  is the longest side and if  $(AC)^2 \neq (AB)^2 + (BC)^2$ , then m ( $\angle$  B)  $\neq$  90° and the triangle is not right-angled.

# Complete each of the following:

(Line Complete and show which of the following triangles is a right-angled triangle :

1

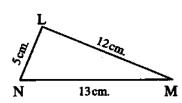


$$(DF)^2 = \cdots$$

$$(DE)^2 + (EF)^2 = \cdots$$

... The triangle is .........

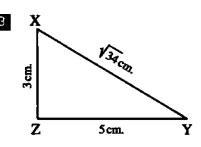
2



$$(MN)^2 = \cdots$$

$$(ML)^2 + (NL)^2 = \cdots$$

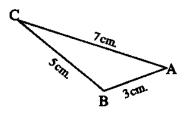
.. The triangle is ......



$$(XY)^2 = (\sqrt{34})^2 = \cdots$$

$$(\mathbf{YZ})^2 + (\mathbf{ZX})^2 = \cdots$$

:. The triangle is .....



$$(AC)^2 = \cdots$$

$$(AB)^2 + (BC)^2 = \cdots$$

:. The triangle is ......

# Homework

## In each of the following figures

Prove that :  $m (\angle B) = 90^{\circ}$ 

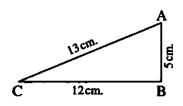


Fig. (1)

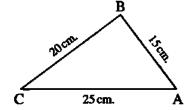


Fig. (2)

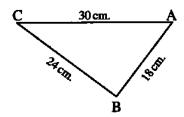


Fig. (3)

# Essay problems:

	In the opposite figure: $m (\angle B) = 90^{\circ}$ , $AB = 4 \text{ cm.}$ , $BC = 3 \text{ cm.}$ AD = 13  cm. and $DC = 12  cm.Find: The area of the figure ABCD « 24)$	· cm² » D	Harm B. C.
١.			
		• • • • • • • • • • • • • • • • • • • •	•••••
		•••••	
			•••••
		•••••	
	In the opposite figure :	D	25 cm. A
	ABCD is a quadrilateral in which: $m (\angle B) = 90^{\circ}$ ,		
	AB = 12  cm., $BC = 16  cm.$ , $CD = 15  cm.$ and $DA = 25  cm$	. Jeg	12 cm.
	<b>1</b> Find: The length of $\overline{AC}$	ò	16cm. B
	Prove that : $m (\angle ACD) = 90^{\circ}$		
			« 20 cm. »
			« 20 cm. »
			« 20 cm. »
۲.			« 20 cm. »
۲.			« 20 cm. »
۲.			« 20 cm. »
۲.			« 20 cm. »
۲.			« 20 cm. »
۲.			« 20 cm. »
۲.			« 20 cm. »

# Homework

	In the opposite figure :
	ABCD is a quadrilateral in which: $m (\angle ABC) = 90^{\circ}$ ,
	ABCD is a quadrilateral in which: $m (\angle ABC) = 90^{\circ}$ , AB = 7  cm., $BC = 24  cm.$ , $CD = 20  cm.$ and $DA = 15  cm.$
	Prove that: m ( $\angle$ ADC) = 90°  C 24 cm. B
w	
٣.	
	ABC is a triangle in which: AB = $4.5 \text{ cm.}$ , BC = $7.5 \text{ cm.}$ , AC = $6 \text{ cm.}$ Prove that: $\triangle$ ABC is right-angled.
٤.	

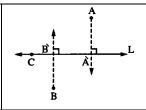
	In the opposite figure :
	ABCD is a quadrilateral in which: $m (\angle B) = 90^{\circ}$ ,
	AB = 9  cm., $BC = 12  cm.$ ,
	CD = 17 cm, and $DA = 8$ cm.
	Prove that: $m (\angle DAC) = 90^{\circ}$ ,
	then find: The area of the figure ABCD « 114 cm <sup>2</sup> . » C 12cm. B
٥.	



# Projections

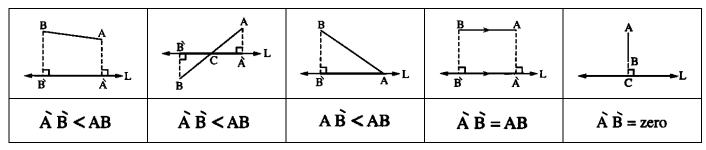
## The projection of a point on a straight line:

- The projection of a point on a straight line is the point of intersection of the perpendicular segment from this point and the straight line.
- 2 If the point lies on the straight line, its projection on it is the same point.



## The projection of a line segment on a straight line:

The projection of a line segment on a given straight line is the line segment whose two endpoints are the projections of the two endpoints of the main line segment on this straight line.



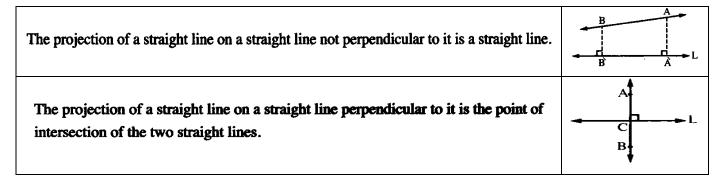
- From the table, we notice that : -

The length of the projection of a line segment on a given straight line (s) the length of the line segment.

## The projection of a ray on a straight line:

The projection of a ray on a straight line not perpendicular to it is a ray straight line.	this
The projection of a ray on a straight line perpendicular to it is a point belon to the straight line.	ging B A C L

# The projection of a straight line on another straight line:



# Complete each of the following:

## Example 1

١.

### In the opposite figure:

 $\triangle$  ABC is right-angled at A and  $\overline{AD} \perp \overline{BC}$ 

### Complete the following:

- 7 The projection of  $\overrightarrow{AB}$  on  $\overrightarrow{AD}$  is ......
- If  $X \in \overrightarrow{AB}$ , then the projection of X on  $\overrightarrow{AB}$  is .......
- If  $\overline{AB} \perp \overline{BC}$ , then the projection of  $\overline{AB}$  on  $\overline{BC}$  is ........
- In  $\triangle$  ABC, if m ( $\angle$ B) = 90°, then the projection of C on  $\overrightarrow{AB}$  is ......
- ABC is a right-angled triangle at A, then the projection of BA on AC is ......

#### Homework

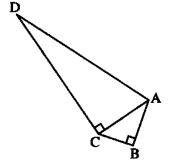
### In the opposite figure :

 $m(\angle B) = m (\angle ACD) = 90^{\circ}$ 

### **Complete:**

٦.

- 1 The projection of  $\overrightarrow{AD}$  on  $\overrightarrow{CD}$  is .......
- 2 The projection of  $\overrightarrow{AC}$  on  $\overrightarrow{CD}$  is .......
- 3 The projection of  $\overline{AC}$  on  $\overline{AB}$  is .......



$\boldsymbol{C}$	hoose the	correct answer	r:	
	The projection	n of a ray on a straight lin	e not perpendicul	lar to it is
١.	(a) a point.	(b) a line segment.	(c) a ray.	(d) a straight line.
۲.		the projection of a line se ine segment itself.	gment on a given	straight line the
	(a) ≤	(b) >	(c) ≥	(d) =
	The length of	the projection of a line se	gment on a straig	tht line parallel to it
٣.	the length of the	he main line segment.		
	(a) <	(b) >	(c) =	(d) ≠
	The length of the	he projection of a line segm	nent on a straight li	ne perpendicular to it is
	(a) greater tha	n the length of the main l	ine segment.	
٤.	(b) equal to th	e length of the main line	segment.	
	(c) greater tha	n or equal to the length o	f the main line se	gment.
	(d) equal to ze	ero.		
	1			
		Home	work	
	The projection	n of a point on a given str	raight line is	•••
٥.	(a) a point.	(b) a line segment.	(c) a ray.	(d) a straight line.
	The projection	n of a line segment on a s	traight line not po	erpendicular to it is
٦.	(a) a ray.	(b) a point.	(c) a line segr	nent. (d) a straight line.

The projection of a line segment on a straight line perpendicular to it is ......

(c) a ray.

(d) a straight line.

(b) a line segment.

(a) a point.

# Essay problems:

$\square$ In the opposite figure: ABCD is a trapezium in which $\overline{AD} /   \overline{BC}$ and m ( $\angle$ ABC	C) = 90° C
If $AD = 9 \text{ cm.}$ , $DC = 10 \text{ cm.}$ and $CB = 15 \text{ cm.}$	<u>5</u>
Find:	±
The length of the projection of $\overline{DC}$ on $\overline{BC}$	B A
The length of the projection of $\overline{DC}$ on $\overline{AB}$	« 6 cm. » 8 cm. »
I t a va a va a vala	
Homework	
☐ In the opposite figure :	D A
$\overrightarrow{AD}$ // $\overrightarrow{BC}$ , $\overrightarrow{AB} = 13$ cm., $\overrightarrow{BC} = 5$ cm.,	
$CD = 15$ cm. and m ( $\angle$ ACB) = m ( $\angle$ DAC) = 90°	
Find:	'\\
The length of the projection of $\overrightarrow{AB}$ on $\overrightarrow{AC}$	C 5cm, B
The length of the projection of $\overrightarrow{CD}$ on $\overrightarrow{AD}$	« 12 cm. • 9 cm. »
The length of the projection of $\overrightarrow{CD}$ on $\overrightarrow{AD}$	« 12 cm. » 9 cm. »
The length of the projection of $\overline{CD}$ on $\overline{AD}$	« 12 cm. » 9 cm. »
The length of the projection of $\overline{CD}$ on $\overline{AD}$	« 12 cm. » 9 cm. »
The length of the projection of $\overline{CD}$ on $\overline{AD}$	« 12 cm. » 9 cm. »
The length of the projection of $\overline{CD}$ on $\overline{AD}$	« 12 cm. » 9 cm. »
The length of the projection of $\overline{CD}$ on $\overline{AD}$	« 12 cm. » 9 cm. »

# Lesson (20) Euclidean Theorem

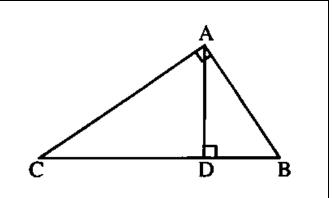
In the right-angled triangle, the area of the square on a side of the right angle is equal to the area of the rectangle whose dimensions are the length of the projection of this side on the hypotenuse and the length of the hypotenuse.

$$(AB)^2 = DB \times BC$$

$$(AC)^2 = DC \times BC$$

$$(DA)^2 = DB \times DC$$

$$DA = \frac{BA \times AC}{BC}$$



# Complete each of the following:

### In the opposite figure:

 $\triangle$  ABC is right-angled at A,  $\overline{AD} \perp \overline{BC}$ 

Complete each of the following:

$$(AC)^2 = \cdots + \cdots$$

$$(AC)^2 = \cdots \times \cdots$$

$$5 \text{ AC} \times \text{AB} = \cdots \times \cdots$$



$$4 (AD)^2 = \cdots \times \cdots$$

 $^{2}$  AD = ..... cm.

## Homework

#### In the opposite figure :

ABC is a triangle in which m ( $\angle$  ABC) = 90°, AB = 4 cm.,

$$AC = 5$$
 cm. and  $\overline{BD} \perp \overline{AC}$ 

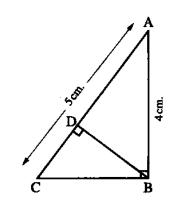
#### Complete:

١.

۲.

$$\mathbf{BC} = \cdots \cdots \mathbf{cm}$$
.

The area of 
$$\triangle$$
 DBC = ......... cm<sup>2</sup>.



# Essay problems:

m	In	the	ani	nosite	figure	•
		ш	vpi	MILL	HEUL	•

ABC is a triangle in which m ( $\angle$  BAC) = 90°,  $\overline{AD} \perp \overline{BC}$ , AB = 8 cm. and AC = 6 cm.

	Scal	
C I	<u> </u>	B
« 6.4 cm. • 3.6		

Find: BD, CD and AD

١.

۲.

•	•	•	•		•	•		•	•		•	•				•	•		•	•					•		
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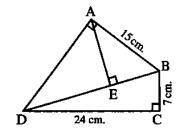
## $\square$ In the opposite figure :

ABCD is a quadrilateral where

$$m (\angle BCD) = m (\angle BAD) = 90^{\circ}$$
,

$$\overline{AE} \perp \overline{BD}$$
, BC = 7 cm., CD = 24 cm.

and AB = 15 cm.



**Find:** 1 The length of each of  $\overline{BD}$  and  $\overline{AD}$ 

The length of the projection of  $\overrightarrow{AB}$  on  $\overrightarrow{BD}$ 

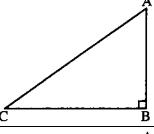
The length of the projection of $\overrightarrow{AD}$ on $\overrightarrow{AE}$	« 25 cm. • 20 cm. • 9 cm. • 12 cm. •

 	• • • • • • • • • • • • • • • • • • • •	

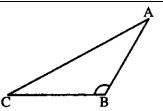
In the opposite figure: $\triangle$ ABC is right-angled at B $ \sqrt{DE} \perp \overline{AC} $ $ \sqrt{AB} = 6 \text{ cm} $ $ \sqrt{ED} = 3 \text{ cm} $ and $ CD = 5 \text{ cm} $ Prove that: $ \Delta CED \sim \Delta CBA $ and find: The length of $ \overline{AC} $	E Scm. D B
and the length of the projection of $\overrightarrow{AB}$ on $\overrightarrow{AC}$	« 10 cm. • 3.6 cm. »
Homework	
In the opposite figure: $\triangle$ ABC is right-angled at B and $\overline{BD} \perp \overline{AC}$ If AD = 4.5 cm. and DC = 8 cm.,  find: The length of each of $\overline{AB}$ , $\overline{BC}$ and $\overline{BD}$	8cm. D 4.5cm. « 7.5 cm. > 10 cm. > 6 cm.

# Classifying triangles according to their angles

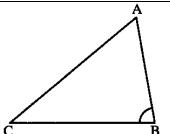
If $(AC)^2 = (AB)^2 + (BC)^2$ , then m $(\angle ABC) = 90^\circ$
and ABC is a right-angled triangle.



If  $(AC)^2 > (AB)^2 + (BC)^2$ , then m ( $\angle ABC$ ) > 90° and ABC is an obtuse-angled triangle.



If  $(AC)^2 < (AB)^2 + (BC)^2$ , then m ( $\angle$  ABC) < 90° and ABC is an acute-angled triangle.



# Complete each of the following:

In $\triangle$ ABC, if $(AB)^2$	$^{2} = (AC)^{2} - (BC)^{2}$	then ∠ C is ·······
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In 
$$\triangle$$
 ABC, if  $(AC)^2 - (AB)^2 = (BC)^2 - 3$ , then  $\triangle$  B is ......

In 
$$\triangle$$
 ABC, if  $(AB)^2 + (BC)^2 = 48 \text{ cm}^2$ ,  $AC = 7 \text{ cm}$ ., then  $\angle$  B is ......

In 
$$\triangle$$
 XYZ, if 90° < m ( $\triangle$  Y) < 180°, then (XZ)<sup>2</sup>.....(XY)<sup>2</sup> + (YZ)<sup>2</sup>

°. If 
$$\angle$$
 A complements  $\angle$  B in  $\triangle$  ABC, then  $(AB)^2 \cdots (AC)^2 + (BC)^2$ 

$$\Delta$$
 ABC is similar to the triangle XYZ , then  $\Delta$  XYZ is ..... according to its angles.

## Homework

In 
$$\triangle$$
 ABC, if  $(AB)^2 = (BC)^2 + (AC)^2$ , then: m ( $\angle$  ·······) = 90°

In 
$$\triangle$$
 ABC, if  $(AB)^2 < (AC)^2 + (BC)^2$ , then  $\angle$  C is ......

١١.	In $\triangle$ ABC	o if $(AB)^2 + (BC)^2 < (AB)^2$	$(AC)^2$ , then $\angle B$ is	••••		
١٢.	In Δ XYZ,	$\sin(XY)^2 = (YZ)^2 + (Z)^2$	$(ZX)^2$ , then $\angle Z$ is	•••		
۱۳.	In Δ XYZ	$o  ext{ if } (YZ)^2 > (XZ)^2 - (XZ)^2 = (XZ)^2$	$(XY)^2$ , then $\angle Y$ is			
C	hoose th	ne correct ans	swer:			
١.	Atriangle w	Atriangle whose side lengths are: 5 cm, 12 cm and 13 cm. its area = cm <sup>2</sup> .				
	(a) 30	(b) 32.5	(c) 78	(d) 60		
۲.	ABC is an obe equals ··		at A, if $AB = 4 \text{ cm.}$ ,	BC = 7  cm., then A	AC can	
	(a) 5	(b) 6	(c) 7	(d) 8		
<u></u>	ABC is a tria	angle in which : $(BC)^2$ =	$= (AB)^2 + (AC)^2 \cdot m (\angle 1)$	B) = $40^{\circ}$ , then m ( $\angle$	C) =	
٣.	(a) 40°	(b) 50°	(c) 90°	(d) 140°		
	ı	H	lomework			
٤.	ABC is an equals		at B if $AB = 5$ cm. • E	3C = 3  cm., then AC	C can be	
	(a) 4	(b) 5	(c) 7	(d) 8		
٥.	<del></del>	acute-angled triangle i	n which $AB = 6 \text{ cm.}$	BC = 8  cm., then the	e length	
	(a) 2	(b) 6	(c) 10	(d) 14		
$\boldsymbol{E}$	ssay pr	oblems:				
	Identify the	type of $\angle A$ in $\triangle ABC$	if $AB = 6$ cm., $BC = 1$	0  cm. and  AC = 8  cm	n.	
					•••••	
,						
•						
					•••••	

	Identify the type of $\angle$ B in $\triangle$ ABC if AB = 10 cm. $\Rightarrow$ BC = 12 cm. and AC = 15 cm.
۲.	
١.	
	In the opposite figure :
	ABCD is a quadrilateral in which AB = 8 cm.,
	BC = 9 cm., CD = 12 cm., AD = 17 cm.
	and $\overline{DB} \perp \overline{AB}$
	1 Find the length of the projection of $\overline{AD}$ on $\overline{BD}$
	Determine the type of $\triangle$ BCD according to its angles.   C « 15 cm. »
, u	
٣.	

	Homework
	Identify the type of $\angle Y$ in $\triangle XYZ$ if $XY = 4$ cm., $YZ = 5$ cm. and $XZ = 7$ cm.
	☐ In the opposite figure :
	ABCD is a parallelogram in which
	BC = 15 cm., CD = 8 cm. and AC = 19 cm.
	Prove that: ∠ ABC is an obtuse angle.  C 15cm. B
•	
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